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On control of swarms in relative configurations: Unified analysis and interactive animation

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***Abstract*— This paper addresses the control and interactive animation of swarms for tasks specified in relative configuration way. Several control goals are of interest: aggregation, formation, and rendezvous. This paper summarizes in a unified framework control systems for achieving these basic objectives. The performance of the corresponding control systems is illustrated by a JAVA-based application developed to visualize the swarm behavior. This is an interactive animation tool which improves the understanding of and intuition for a number of aspects dealing with the swarms tasks. This tool allows directly manipulating graphical representation of the systems such as a choice among formation structures, initial configuration of the swarm, and control parameters, and getting instant feedback on the effects.**

***Index Terms*— Swarm, Agents, Control, Interactive animation, Aggregation, Formation, Rendezvous.**

I. INTRODUCTION

IN the modern technology jargon, the underlying concept of agent deals with an entity having the following features: autonomy, adaptability and mobility. Agents appear in several applications in communication and robotics where the behavior of a large number of agents -a swarm- is of interest. The control of swarms to perform desired tasks is a recent issue in the control and robotics communities [1,2,3]. This paper adopts the definition of agent given in [4]: An agent is a point in the Cartesian plane with no kinematic constraints of motion. More formally, the configuration of agent i is described by $z_i = [x_i y_i]^T \in R^2$ whose dynamics is

$$\dot{z}_i = u_i \tag{1}$$

where $u_i = [u_x i u_y i]^T \in R^2$is the control input vector whose components are the agent velocity along the Cartesian axes.

Consider a swarm of n individuals (agents) denoted by $z_1, \cdots, Z_n$ where we use the notation

$$z_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

for$i = 1, \cdots, n$. The position of all the agents gives the swarm configuration denoted by

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \in R^{2n}$$

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According to (1) the corresponding swarm dynamics can be written as

$$\dot{z} = u \tag{2}$$

where$\bar{u}$ is the control input of the swarm:

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \in R^{2n}$$

It is convenient to define the center of the swarm as

$$\bar{z} = \frac{1}{n} \sum_{i=1}^{n} z_i$$

Define the size of the swarm as the radius $\varepsilon > 0$ of the hyperball $B_\varepsilon(\bar{z})$ centered at $\bar{z}$

$$B_\varepsilon(\bar{z}) = \{z \in R^{2n} : \| \bar{z} - z \| \leq \varepsilon\}$$

A number of desired tasks can be formulated for swarms [2]. This paper focuses on tasks dealing with relative configuration among their agents where there is no information on the absolute location neither of the agents nor their desired locations. Three basic tasks arise in this context: aggregation, formation, and rendezvous. The objective of this paper is to summarize recent control strategies reported in the literature but under unified notation and framework, and to illustrate them trough interactive animations.

Advances in software and hardware have recently allowed designing tools with much better man-machine interaction, with intuitive graphical user interfaces. Packages for multi-agents modeling and simulation of complex systems with these features are now available. Java-based software like SWARM (http://www.swarm.org) and NetLogo [5] belong to these packages.

Although SWARM and NetLogo are useful tools for researchers in the study of agent based models, they are not well suited for simulation of dynamic systems characterized by Ordinary Differential Equations (simple numerical methods are invoked), and more important perhaps, they have limited interactivity. Interactiveness relies on the display of an animated graphical representation of the physical system where the simulation and system parameters may be changed at any time and the user may interact by dragging graphical objects during simulation. This feature, exploited by the users' visual feedback, creates an environment for analysis of relevant parameters and emulation of situations that may give valuable insight into phenomena of great complexity [6].

A general-purpose interactive animation tool is Easy Java

Simulations (EJS), a free of charge software, valuable to help create interactive simulations and animations in the Java language [7]. This software tool is target for both science and engineering students and teachers with basic programming skills but who cannot afford the big investment of time needed to create a complete graphical simulation; a summary of EJS and its application in automatic control is given in [8].

Dynamic systems characterized by Ordinary Differential Equations (ODE) can be simulated by EJS. These equations are described within EJS by using self-contained facilities (ODE solvers and Java code editor). The equations of the physical laws that rule the phenomenon under study are written in a way similar that written on a blackboard. EJS provides facilities to declare the system variables, the initial conditions, and the way how the user interacts with the system and modifies the variable values.

EJS provides also a catalog of graphical elements to help the user to create views of the simulated dynamic system for animation and continuous visualization that reacts to the user interaction. These elements include containers, buttons, text fields, boxes, sliders, etc. as well as elements for designing the view of the physical phenomenon like particles, bodies, vectors, etc. These graphical elements can be used to build the graphical interface just by drag-and-drop the mouse.

A contribution of this paper is the application of interactive animation tools to illustrate the control of swarms. These tools are focused on objects to explore different views of the systems, changing the initial configuration of the swarms and immediately see the consequences on the system behavior. We have found that the tool is a valuable complement to understand the control of swarm because the high degree of interactivity makes the tool stimulating and quickly captures the interest of the user.

II. AGGREGATION

In the aggregation of swarms, the agents have to maintain a cohesive behavior, for instance, by converging to a bounded domain. Given a desired size of the swarm ε, the aggregation objective can be stated formally as

$$\lim_{t\to\infty} dist(z(t), B_\varepsilon(\bar{z}(t))) = 0 \tag{3}$$

where $dist(z(t), B_\varepsilon(\bar{z}(t)))$ denotes the smallest distance from z to any point in the set $B_(\bar{z})$.This means that all agents converge asymptotically to the hyperball centered at the swarm center with radius ε .In this section it is assumed limited sensor capabilities in the sense that there is no way to measure the agents absolute position z_i, although their relative position $z_i - z_j$ is available. The information consensus under dynamically changing interaction topologies to represent information exchanges among multiple agents is addressed in [9].

A control scheme to resolve the aggregation of swarms was recently introduced in [10] inspired from the potential field method widely used for autonomous mobile robot navigation [11,12]. We summarize this approach right away.

Depending on the formation goal, the potential field method requires the design of a proper artificial potential function $U : D \longrightarrow R$ where $D \subset R^{2n}$. It is assumed that $U(z)$ is twice continuously differentiable in D. Once this function is available, based in the gradient descent approach, the control law uses the negative of the potential function gradient

$$u = -B(z)\frac{\partial U(z)}{\partial z} \tag{4}$$

where $B(z)$ is a symmetric positive definite matrix. The dynamics of the closed-loop system is obtained by substituting the control law (4) into the swarm model (2). This yield

$$\dot{z} = -B(z)\frac{\partial U(z)}{\partial z} \tag{5}$$

whose structure corresponds to a gradient system. Nice properties of gradient systems such as stability and convergence are presented in [13].

The artificial potential function proposed in [10] is given by

$$U(z) = \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[a \parallel z_i - z_j \parallel^2 + bc\exp(-\frac{\parallel z_i - z_j \parallel^2}{c})]$$

where a,b and c are positive constants, and for a vector $y \in R^n$,its Euclidean norm is given by $\parallel y \parallel = \sqrt{y^T y}$.

Above potential function induces via (4) the following control law

$$u = -\begin{bmatrix} \sum_{j=1,j\neq 1}^{n}[z_1 - z_j][a - b\exp(-\frac{\|z_1 - z_j\|^2}{c})] \\ \sum_{j=1,j\neq 2}^{n}[z_2 - z_j][a - b\exp(-\frac{\|z_2 - z_j\|^2}{c})] \\ \vdots \\ \sum_{j=1,j\neq n}^{n}[z_n - z_j][a - b\exp(-\frac{\|z_n - z_j\|^2}{c})] \end{bmatrix} \tag{6}$$

where for simplicity $B(z) = I$ is the identity matrix.

By invoking Theorem 1 in [10], it can be shown that the aggregation objective (3) is globally reached provided that

- $b > a > 0$
- $c < 2[\frac{a}{b}\varepsilon\exp(\frac{1}{2})]^2$

It is worth to remark that implementation of the control law (6) requires that each agent knows the relative location of the remaining agents in the swarm, but knowledge of their absolute positions is unnecessary.

A. Interactive animation

The graphical interface of the interactive application for swarm aggregation is depicted in Figure 1. For the sake of clarity, the application considers six agents, but the number can be increased. For illustration purpose, the agents are represented graphically by bees or bats according to the user choice.

The interface shows three areas. The main area is on the upper center of the window. This is devoted to present the animation of the swarm motion. Thanks to the mouse, the user can interact on-line by dragging any agents to an arbitrary initial location or to new location without stopping the simulation. Below this area, there is a row containing three buttons: start, pause, and reset. Also, selecting boxes are available for drawing the agent traces, velocity vectors (control actions), and for plotting the norm of each agent with respect to the center of the swarm. Figure 2 shows typical plots where the value of the desired radius is also depicted.

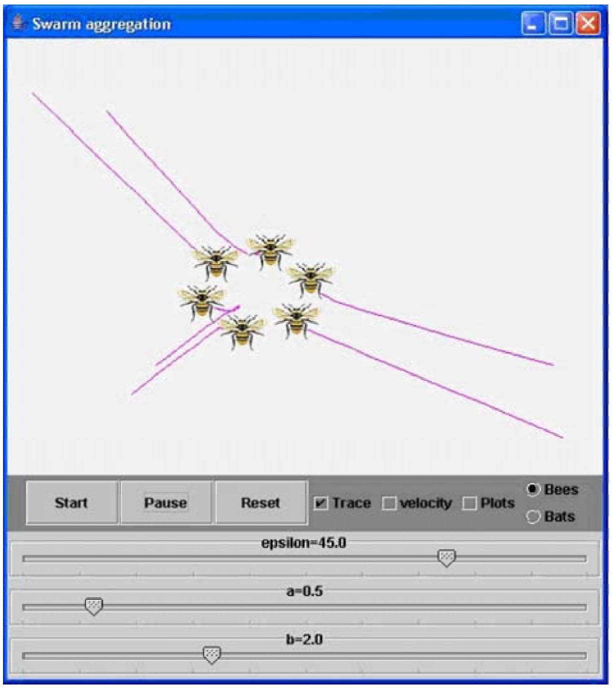

Fig. 1. Swarm aggregation: bees

The plots indicate that after the transient, all agents remain inside the hyperball of radius $\varepsilon = 45$. Finally, on the right part of the row, the user can select the graphical representation of the agents between bees of bats.

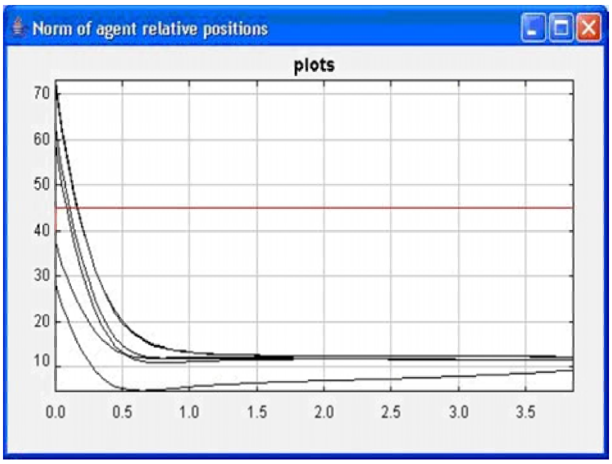

Fig. 2. Swarm aggregation: Plots

The lower part of the graphical interface contains three sliders for specification of the parameters:ε, a, bThe user can interact with these sliders to change the parameter values on-line when the simulation is running and visualize immediately the corresponding system response. Figure 1 shows the trace of the agents for $\varepsilon = 45, a = 0.5, b = 2$. whereas Figure 3, using bats for graphical representation of the agents starting from the same initial locations and desired swarm size, is the evolution with parameters $a = 0.8$ and $b = 5$. Notice that the final distribution of the agents differs from each other.

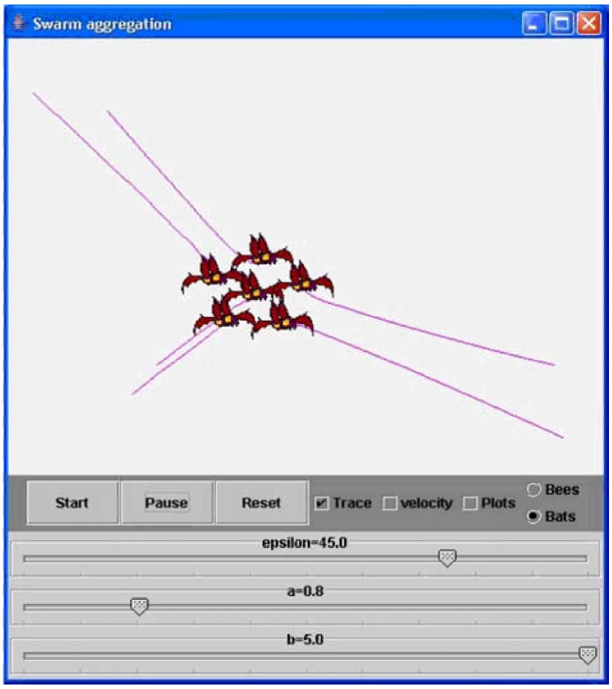

Fig. 3. Swarm aggregation: bats

III. FORMATION

Formation of swarms deals with the problem of controlling the relative position of the agents in the swarm [14]. It can be classified according to a number of factors such as communication links, geometry of the sensory space, absolute desired configuration, and relative desired configuration with invariance to translation and rotation [15]. This section deals with relative desired configuration under invariance to translation but a stronger finite sensor-communication capability, more specifically, each agent knows only the relative location of the next agent, i.e. $z_i - z_{i+1}$ for $i = 1, \cdots, n-1$ and $z_n - z_1$ are available for feedback purpose, but the absolute location z_i cannot be neither measured nor computed.

A geometric shape for a swarm may be specified by formation functions or formation constraints [15,16] in terms of the relative locations or distances between each agent. Let us define the desired relative location between agent i and $i+1$ by $\Delta_i \in R^2$ for $i = 1, \cdots, n-1$ and $\Delta_n \in R^2$ corresponding to the relative location between agent n and agent 1.Given the arbitrary desired relative displacements $\Delta_1, \Delta_2, \cdots, \Delta_n$, the remaining one is computed as $\Delta_n = -\sum_{i=1}^{n-1} \Delta_i$.

In virtue that $\sum_{i=1}^{n} \Delta_i = 0$,then there exist agent configurations that match the formation constraints. For analysis purposes and without loss of generality, choose an arbitrary set of n points $z_1^*, z_2^*, \cdots, z_n^* \in R^2$ satisfying the formation constraint, that is

$$\Delta_i = z_{i+1}^* - z_i^* \tag{7}$$

for $i = 1, \cdots, n-1$ and $\Delta_n = z_1^* - z_n^*$.For the sake of notation define

$$z^* = \begin{bmatrix} z_1^* \\ z_2^* \\ \vdots \\ z_n^* \end{bmatrix}$$

The formation control of a swarm under the objective of relative configuration with invariance to translation attempts to guide the agents to reach a desired geometric form with arbitrary unspecified translation. The formation objective with relative configuration and invariance to translation can be stated formally as

$$\lim_{t\to\infty}[z(t) - z^* - T\xi] = 0 \quad (8)$$

for any $\xi \in R^2$ where

$$\mathrm{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \in R^{2n\times 2} \quad (9)$$

A solution to this formation control of swarms can be adapted following ideas form the cyclic pursuit method reported in [4]. We propose the following control law

$$u = kA[z - z^*] \quad (10)$$

where k¿0 is a control gain, and $A \in R^{2n\times 2n}$is defined by

$$A = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & -1 \end{bmatrix}$$

It is important to observe that for implementation purpose, the control law (10) produces the control actions

$$\begin{aligned} u_i &= k[z_{i+1} - z_i - k[z_{i+1}^* - z_i^*], \\ &= k[z_{i+1} - z_i] - k\Delta_i \end{aligned}$$

for $i = 1, \cdots, n-1$ and $u_n = k[z_1 - zn] - k\Delta_n$. We emphasize that implementation of the control law needs only information about the relative location between each agent and the next one in the order, as well as the formation constraints Δ_i", but information about absolute -actual or desired- configuration of the swarm is unnecessary.

It is worth noticing that matrix A is negative semidefinite but a singular one whose null space $N(A)$ is given by

$$N(A) = \{T\xi, \forall\xi \in \ R^2\}$$

.

The closed-loop dynamics is obtained by plugging the control law (10) into the swarm model (1); this leads to

$$\dot{z} = kA[z - z^*] \quad (11)$$

Since A is singular, then above equation has an infinity number of equilibria given by

$$z^* + T\xi, \forall\xi \in R^2. \quad (12)$$

The behavior of the closed-loop system (11), which is an autonomous one, can be studied by invoking the LaSalle's invariance principle [13]. To this end, consider the following nonnegative differentiable function

$$V(z) = \frac{1}{2} \parallel z - z^* \parallel^2$$

Its time derivative along the trajectories of the closed-loop system (11) yields

$$\dot{V}(z) = k[z - z^*]^T \cdot A[z - z^*] \quad (13)$$

which satisfies $\dot{V}(z) \leq 0$ for all $z \in R^2$ in virtue that $k > 0$ and A is a negative semidefinite matrix. This is a first requirement of the LaSalle's invariance principle. Then, it follows that the system trajectory $z(t)$ tends to the largest invariant set contained in the following domain:

$$\{z \in R^{2n} : \dot{V}(z) = 0\}$$

. But from (13) it results that this domain is equivalent to

$$\{z \in R^{2n} : A[z - z^*] = 0\}$$

.which is exactly the equilibria set (12). This is an invariant set, and therefore the largest invariant one; thus, the LaSalle's invariance principle ensures that the system trajectories tend to the equilibria set, so the control objective (8) is attained.

A. *Interactive animation*

The interactive animation application of swarm formation is illustrated by six agents. The interactive tool allows selecting between four formations: triangle, rectangle, line, and point. Each formation corresponds to a particular choice of vector z*in the control law (10). The graphical interface (see Figures 4 and 5) allows the user to select the desired formation by clicking the corresponding selection box. The graphical illustration of the agents can also be chosen between bees or bats as well as the agents traces and velocities (control actions). Finally, at the lower right side of the interface there is the control gain k slider. The user can interact with all these elements and drag the agents to new locations on-line and visualize immediately the effect on the whole system behavior.

Figures 4 and 5 depict two views: the rectangle and triangle formations, respectively. Each scene corresponds to the same arbitrary initial configuration. The traces of the agent motion -bees and bats, respectively- show that the agents reach asymptotically places with a rectangle and triangle shape as desired, respectively.

IV. RENDEZVOUS

In the context of motion coordination, the swarm rendezvous objective (also termed as agreement objective) is to cause all agents of the swarm to eventually rendezvous at a

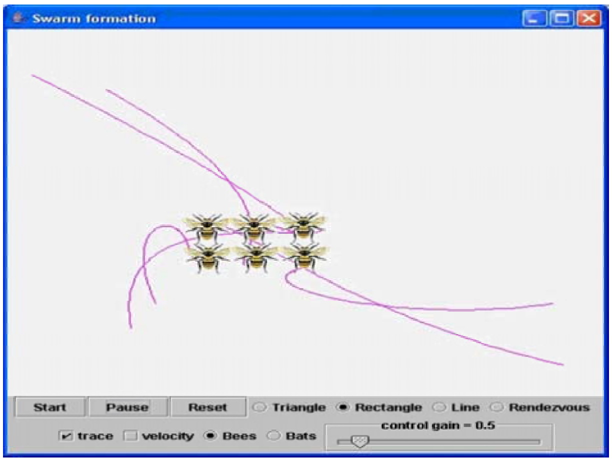

Fig. 4. Swarm formation: Rectangle

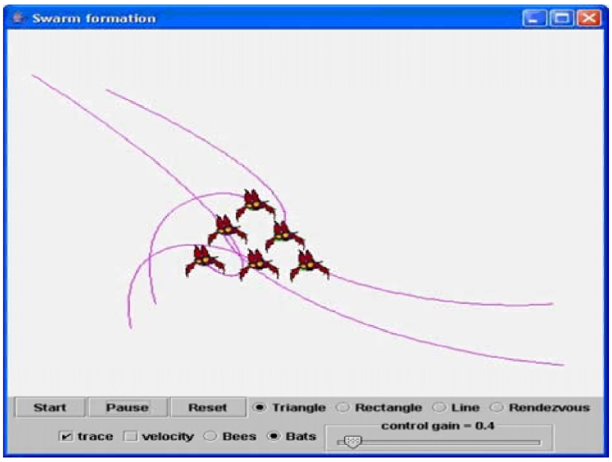

Fig. 5. Swarm formation interface: Triangle

single unspecified location, that is, to steer each agent to a common location [3,17].

Formally, the rendezvous objective is given by

$$\lim_{t\to\infty}[z(t) - T\xi] = 0 \tag{14}$$

for any $\xi \in R^2$, where T as defined in (9).This is equivalent to state that all agents reach asymptotically the same location ξ, that is

$$\lim_{t\to\infty} z_i(t) = \xi$$

, for $i = 1, \cdots, n$.

Regarding the information flow among agents of the swarm, this section considers two scenarios: first, each agent knows the relative location of the remaining members of the swarms $z_i - z_j$, and second, a stronger one where each agent knows only the relative location of the next one in the order $z_i - zi + 1$.

A. Rendezvous by aggregation

Under the scenario where each agent knows the relative location of the remaining ones in the swarm, a solution for the rendezvous objective can be borrowed from the aggregation objective (3) by decreasing the desired size ξ of the swarm. At the limit as $\xi \to 0$, the parameter c also vanishes, hence this procedure reduces the control law (6) to

$$u = -a \begin{bmatrix} \sum_{j=1,j\neq 1}^{n}[z_1 - z_j] \\ \sum_{j=1,j\neq 2}^{n}[z_2 - z_j] \\ \vdots \\ \sum_{j=1,j\neq n}^{n}[z_n - z_j] \end{bmatrix} \tag{15}$$

where a is a positive parameter. This control law can be also rewritten as

$$u = -a \begin{bmatrix} nI_2 & -I_2 & -I_2 & \cdots & -I_2 & -I_2 \\ -I_2 & nI_2 & -I_2 & \cdots & -I_2 & -I_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -I_2 & -I_2 & -I_2 & \cdots & nI_2 & -I_2 \\ -I_2 & -I_2 & -I_2 & \cdots & -I_2 & nI_2 \end{bmatrix} z$$

where I_2 is the 2-dimension identity matrix. By substituting the control law (15) into the swarm dynamics (1) it results the following closed-loop system equations

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_n \end{bmatrix} = \begin{bmatrix} [n-1]z_1 - [z_2 + z_3 + \cdots + z_n] \\ [n-1]\, z_2 - [z_1 + z_3 + \cdots + z_n] \\ \vdots \\ [n-1]\, z_n - [z_1 + z_2 + \cdots + z_{n-1}] \end{bmatrix}$$

It follows that the dynamic behavior of the swarm center $\bar{z}$ is described by

$$\dot{\bar{z}} = \frac{1}{n}\sum_{i=1}^{n} \dot{z}_i = 0$$

which implies that the center is stationary. Therefore, all agents converge to the center of the initial configuration of the swarm, i.e. the control objective (14) is achieved with $\xi = \bar{a}(0)$.

B. Rendezvous by formation

In the scenario where each agent knows the relative location of the next one in the order, the previous approach can not be utilized because it needs the information about the relative location each agent and the remaining individuals of the swarm for computation of the control law (15).

For this situation, the formation strategy based in cyclic pursuit can be a solution because by choosing $z^* = 0 \in R^{2n}$ the formation control objective (8) reduces to the rendezvous objective (14). Therefore, for the rendezvous objective, the formation control law (10) becomes [4]

$$u = kAz$$

The closed-loop dynamics obtained by substituting the control law into the swarm model (1) leads to $u = kAz$. The behavior of each agent in the swarm is governed by

$$\dot{z}_i = k[z_{i+1} - zi]$$

and $\dot{z}_n = k[z_1 - z_n]$. From this information it can be easily shown that the center of the swarm $\bar{z}$ satisfies $\dot{\bar{z}} = 0$, this means that the center is stationary for all time. Because all agents converge to a point, then we have the conclusion that the swarm converge to the center of the initial configuration

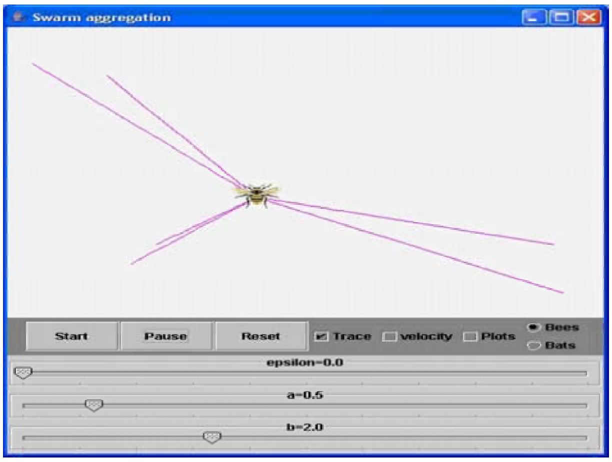

Fig. 6. Swarm rendezvous by aggregation

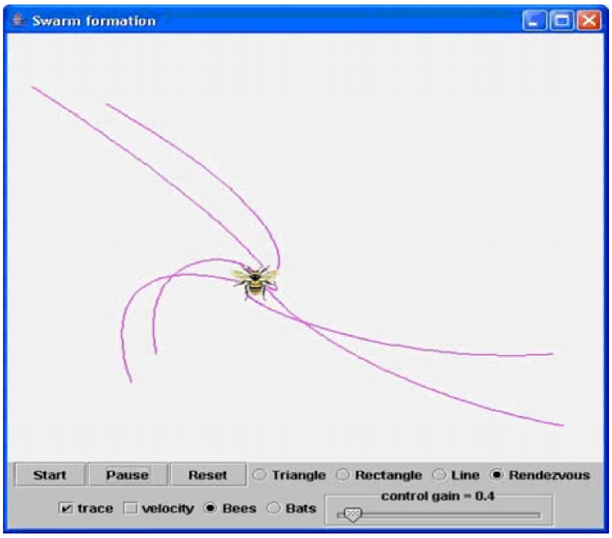

Fig. 7. Swarm rendezvous by formation

of the swarm, i.e. the rendezvous objective (14) is guaranteed with $\xi = \bar{z}_0$.

C. Interactive animation

The control strategies for swarm rendezvous previously presented are special cases of swarm aggregation and formation. Therefore, the interactive animation for swarm rendezvous reduces also as particular cases of the developed interactive applications for aggregation and formation.

More specifically, the swarm rendezvous from aggregation can be obtained by decreasing the size ε . The swarm behavior is shown in Figure 6 where $\varepsilon = 0.0$. As expected, all members of the swarm converge to the same point. The interactive animation for rendezvous by formation arises for the choice of the selection box "rendezvous" in the graphical interface depicted in Figure 7. This corresponds to $z^* = 0 \in R^{2n}$ which induces a zero relative location between any two adjacent agents. It can be seen from the agent traces in Figure 7 that after a transient all agents move toward a common point.

V. CONCLUDING REMARKS

The control of swarms in robotics offers challenges from theoretical and practical points of view. Although a number of control objectives can be formulated for swarms, this paper has addressed those in relative configurations where the absolute locations for the agents or their absolute desired locations are unnecessary (or unavailable due to possible limited-range sensor and communication capabilities). The paper has summarized in a unified framework the three basic ones: aggregation, formation, and rendezvous.

As a pedagogical element, the paper has exploited the Java-based open-source software EJS to design graphical interfaces illustrating the concepts of each control formulation and to visualize the whole swarm behavior. These applications are a valuable complement to understand the swarm control objectives because the high degree of interactivity makes the applications stimulating and quickly capture the interest of the users.

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