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A Biologically Inspired Controller for Swarms in Dynamic Environments

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***Abstract*— In this paper we present a biologically inspired nonlinear controller for swarms flocking in dynamic environments. Based on some observations in natural swarming phenomena and certain hypothesis proposed by biologists, we present a general decentralized controller to make swarm members flock together, which can be applied into certain related applications such as robot swarm or mobile sensor networks. We assume that during the process of flocking, each swarm member can sense and interact with its nearest neighbors while following certain path clues from the potential profile of the environment. With a new set of mutual interaction functions and the assumption of strongly connected graph, the controller is proved to enable the velocities of all swarm members to converge to a common value with bounded errors. The advantage of this controller is that only local and relative information are needed to achieve stable group behavior for either fixed or dynamic swarm topology. In addition, the topological graph of the swarm considered in the controller is the sensing rather than the communication graph. As a result, the communication module is not needed for the implementation of the agents in engineering. We also discuss the convergence time of the swarm for one dimensional case. A few sets of simulations, including the case of agents being lost during swarm's motion, are presented to verify the feasibility of the proposed controller.**

***Index Terms*— Swarm robotics, decentralized control, biologically inspired controller.**

I. INTRODUCTION

OVER the past decades the natural phenomena of swarming have invoked extensive research interests in diverse fields of scientific and engineering disciplines. Some interesting examples of such phenomena can be observed in creatures such as migrate birds, tuna fish, bees, and by flocking, schooling and herding. The collective group behaviors are believed to have certain advantages over individual ones, for example, increasing the survival chances for the whole group under the danger from predators. The inspiring point in these phenomena is that although the intelligence of the individual member is limited, the sophisticated and efficient group behavior can still be achieved.

Many research efforts in different areas, from biological science to systems and controls, and from physics to computer science, have been made to explain how the creatures form a collective swarming behavior without a global coordinator. Biologists have observed and analyzed the swarm behaviors of different species for decades [1]-[9]. Some interesting phenomena were first observed and recorded by them. For example, Miller *et al.* found that a swarm of sandhill cranes can flock together in a same speed with a constant spacing (about 5.8ft) [7], and tuna shoals are observed to school together with a separation of 0.16-0.25 body length in shapes of 1D "Soldier", 2D "surface", and 3D "ball" [3].

In most biological systems studied so far, the high-level group behavior has been generally assumed to be mediated by small sets of simple low-level interactions between individuals and the environment, and among swarm members [1][2][5][6]. Several mathematical models have been presented to address how the social group is organized and how the structure of the swarm changes with different environments. The approaches involved basically include Eulerian and Lagrangian models, in both of which the model could be either deterministic or stochastic/statistical. The Eulerian approach applies nonlinear partial differential equations to describe the evolving swarm density [1][8]; the Lagrangian model is individual-based and typically derived by the classical Newtonian mechanics law for the motion of each swarm member[2][5][6]. In parallel to the deterministic models by Eulerian or Lagrangian approaches, the stochastic and statistical methods consider the fluctuations of individual states and overall behaviors of the swarm, respectively [4].

Besides mathematical biologists, physicists also produced plenty research results on self organizations of swarms of self-propelled particles [11]-[15]. In [14] Vicsek *et al.* presented a simulation model based on nearest neighborhood law, in which each particle's heading is updated by the average of the headings of its nearest neighbors and itself. Simulations in [14] indicate that the headings of all particles can converge to a common value. A behavior based simulation model was proposed by Craig Reynolds for animating the coordinated motions of a group of agents [16]. He named the simulated generic entity as "boids" [16][17]. Although they are developed independently, it turns out that Vicsek's model is a special case of boids.

To mathematically prove Vicsek's results, Jadbabaie *et al.* presented a discrete kinematic model and a decentralized controller by nearest neighborhood law in [18]. Some mathematic tools from algebraic graph theory are utilized in [18] to prove the convergence of all swarm members' velocities. In their later work [19]-[21], a continuous dynamic model and a decentralized controller are proposed for fixed and dynamic swarm topologies. The controller includes heading and velocity adjustment components, which are based on

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nearest neighbors' states. By discontinuous stability theory, the controller is proved to cause all members' headings to converge to a same value, and all velocities to become same for dynamic topology. Further theoretical extensions of this work were presented in [28]. It is shown that consensus is achieved asymptotically if the union of the "information exchange graph" is connected most the time as the system evolves.

In [23] Gazi and Passino presented a continuous first-order kinematic model for the swarm members, and propose a decentralized controller for the analysis of swarm aggregation in n-dimensional space. The author showed that the individuals can form a cohesive swarm in a finite time. An explicit bound of the swarm size is also derived in it. The results in [23] are extended to a more general class of virtual force functions in [24]. In their later work [25]-[27], the same approach is exploited to demonstrate the collective behavior of swarms moving in certain environments. In [22], Liu *et al.* used a second-order dynamic model to study the stable foraging of swarms in certain environments with noise. However, all the controllers proposed in [22]-[27] require each member to know all other swarm members' states, which is impossible for the natural creatures.

In [29] a systematical framework is presented for the average consensus problem in networks of agents with fixed and switching topology and communication delays. With a simple single integrator model for each agent, the authors discussed average consensus problems for directed networks with fixed topology, directed networks with switching topology, and undirected networks with communication time delay and fixed topology. Moreover, a disagreement function is introduced for disagreement dynamics of directed networks with switching topology. The undirected networks case is discussed by the same authors in [30]. Based on these, the authors developed flocking algorithms with guaranteed convergence and the capability of dealing with obstacles and adversarial agents [31][32]. In [31][32], it is demonstrated that flocks are networks of dynamic systems with a dynamic topology. This topology is a proximity graph that depends on the state of all agents and is determined locally for each agent, i.e. the topology of flocks is a state-dependent graph.

In this paper we study the natural phenomena of swarms flocking in dynamic environments, such as tuna fish school in ocean and migrate birds flock from the north to the south. Based on some biological observations and hypothesis that the swarm members can sense and follow certain path clues from the potential profile of the environment individually and at the same time interact with its nearest neighbors via virtual force, we propose a general decentralized controller which needs only local and relative information to make swarm members flock together, which can be applied into certain related applications such as robot swarm or mobile sensor network. With a new set of mutual interaction functions, this decentralized controller can enable the velocities of all swarm members to converge to a common value with bounded errors. Compared with the controllers proposed in [19]-[21], which also only use local information from nearest neighbors, we prove the velocity convergence in a different manner for both fixed and dynamic swarm topologies. The advantage of the controller is that the global position sensing device and communication module are not needed.

This paper is organized as follows. In section II an n-dimensional dynamic model for individual swarm member is presented. In section III we illustrate the ideas of virtual force and nearest neighborhood law, and present a general decentralized controller with a new specific virtual force function. The convergence of the velocities of all swarm members are also discussed in section III. A few sets of simulation results including the case of agents being lost during swarming are presented in section IV to verify the proposed controller. This paper ends with some conclusions and future research topics in section V.

II. Modelling of Swarm Members

In nature there are many species of creatures who demonstrate the self-organized swarming behavior in groups. Such examples can be observed at a variety of size scales, from the macroscopic flocks of birds and schools of fish, to the smaller insect swarms, and even microscopic colonies of bacteria. We use a generic name "agent" to represent the swarm member of arbitrary species in this paper. We study the behavior of the whole swarm through a bottom-up approach, that is, we first investigate the dynamic model and decentralized controller of individual agent, then show the behaviors (velocities) of all agents in the swarm can automatically self-organized into a common value by the proposed controller.

Consider a swarm of N agents moving in an n-dimensional Euclidean space. For simplicity, we ignore each agent's dimension and treat it as a point mass.

For the ith agent in the swarm, it can be modelled as

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= \frac{1}{m_i} u_i \end{aligned} \tag{1}$$

where $i = 1, 2, ..., N$, $x_i \in \mathbb{R}^n$ is its global position relative to ground coordinates, $v_i \in \mathbb{R}^n$ is its velocity, m_i is the mass, and u_i is the control input. We assume the mass m_i's are known and no disturbance forces on each agent. Normally the agents move in a 2-D or 3-D Euclidean space, i.e., $n = 2$ or 3. In this paper, we consider the general n-dimensional case.

Define

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{2}$$

$$\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i \tag{3}$$

in which $\bar{v} \in \mathbb{R}^n$ represents the average velocity of all swarm members. We will show that all agents' velocities can converge to $\bar{v}$ with bounded errors by the decentralized controller proposed in this paper.

Apparently, x_i's are global measurements with respect to certain fixed ground coordinates. For creatures in the nature, however, it is very hard to have such information. For example, each fish in a schooling group of tuna shoals may never know how far it is from the north pole of the earth, but they would rather concern the relative distances between each other. The advantage of the controller proposed in this paper is that only local and relative information is needed to achieve the collective group behavior. Even though the current technology provides state-of-the-art GPS sensor which can provide the global position for each agent, by the controller in this paper, such costly device is no longer needed.

Let $x = [x_1^T, x_2^T, ..., x_N^T]^T$ and $v = [v_1^T, v_2^T, ..., v_N^T]^T$ represent the position and velocity of the whole swarm, respectively.

Define two error states as

$$e_{x_i} = x_i - \bar{x},$$
$$e_{v_i} = v_i - \bar{v}.$$

Clearly,

$$\dot{e}_{x_i} = \dot{x}_i - \dot{\bar{x}} = e_{v_i}, \tag{4}$$
$$\dot{e}_{v_i} = \dot{v}_i - \dot{\bar{v}}. \tag{5}$$

To determine each agent's motion, we only consider its nearest neighbors. This well-known neighborhood law is from the fact that the perception and sensing range of any creature is not unlimited, and has been proposed and utilized for decades. In this paper, we rather consider it as defining the range in which certain swarm mates affect an agent's motion.

Define the spacing between two different agents i and j as $d_{ij} = \|x_i - x_j\|$. Before we proceed, the definition of neighbors is provided as following.

Definition (Neighbors): Two different agents i and j ($i \neq j$) are called *neighbors* of each other if $\|x_i - x_j\| \leq d_0$, in which d_0 is a given positive number.

In this paper d_0 is determined by the sensing range of the agents. For simplicity, we assume that all agents have an identical sensing range, thus use a single d_0 to define neighbors for all agents.

Moreover, define

$$\mathbb{N}_i \triangleq \{j : \|x_i - x_j\| \leq d_0, j \neq i, j = 1, ..., N\} \tag{6}$$

to represent the set of all neighbors of agent i.

III. CONTROLLER AND STABILITY ANALYSIS

In this section, based on the hypothesis of virtual force among swarm members and some biological observations, we propose a decentralized controller for individual agents, which only needs local and relative sensing information, to make all their velocities converge to a common value with bounded errors.

The hypothesis of virtual forces in social creatures has been adopted in mathematical biology community for a long time [2][4]-[6]. It assumes that the swarm members interact with each other through certain virtual forces. The virtual forces (repulsive or attractive forces) drag or push agents from each other, which subsequently affect individual velocity and position.

Moreover, since the agents (of any creature) have limited perception ranges, we also assume that it can only sense virtual forces from certain neighbors. In other words, the motion of each agent is assumed to be affected only by the virtual forces from its neighbors within a certain range, and all forces from other agents beyond this range will be ignored. Note that, for the same reason of the limited sensing range in both nature and engineering, it is impossible for each agent to know all other swarm-mates' states. To make the results more realistic and close to the nature, we propose a decentralized controller for each agent that needs no global information of the whole swarm to achieve the group behavior.

On the other hand, although each agent hardly has the full knowledge about the environment, it is reasonable to assume that it knows about the local environment around its current position. This assumption can be justified by the observations in biological systems. For example, the European robins and homing pigeons can sense the magnetic field of the earth to determine their heading directions [9], and some tropical reef fish can perceive and school along the ocean currents [10]. We assume that the swarm moves in an environment with a potential function $J(x)$, the gradient of which at x_i, i.e., $\nabla_{x_i} J(x)$, is known. Also $J(x)$ is assumed to have finite bounded slopes.

Based on the above discussion, we propose a general decentralized controller for each agent as

$$\begin{aligned} u_i = & -m_i k_p \{v_i - \nabla_{x_i} J(x)\} \\ & + m_i \sum_{j \in \mathbb{N}_i} \{f_r(x_i - x_j) - f_a(x_i - x_j)\}, \end{aligned} \tag{7}$$

where k_p is a positive design constant, and f_r, f_a are virtual repulsive and attractive forces, respectively. The implication of the controller is that each agent recognizes and follows the environmental "path clue" individually, and at the same time interacts with its neighbors through virtual forces to adjust its velocity and position.

Note that all the information needed for the above controller are just from a finite range of each agent's neighborhood, and no knowledge of all other swarm-mate's states is required.

Remark 1: Note that the measurements involved in the controller (7) are just relative distances between pairs of agents, i.e., $x_i - x_j$, rather than global positions x_i or x_j. This means that the costly GPS sensor devices can be saved and relative range sensors are sufficient.

Specifically, we select a new set of virtual force functions as

$$\begin{aligned} f_r(x_i - x_j) &= k_r (x_i - x_j) \exp\left(-\frac{\|x_i - x_j\|^2}{c}\right), \\ f_a(x_i - x_j) &= k_a \tanh\left(\frac{x_i - x_j}{2}\right), \end{aligned} \tag{8}$$

where k_r, k_a and c are positive design constants.

The virtual forces are shown in Fig.1 when $k_r = 2.0$, $c = 8e$, and $k_a = 1.5$. The combined effect of the above repulsive and attractive forces is shown in Fig.2.

Note that at a certain distance (about 6.98) in Fig.1, the virtual repulsive and attractive forces are equivalent, and the combined effect is zero. We denote such balance distance as

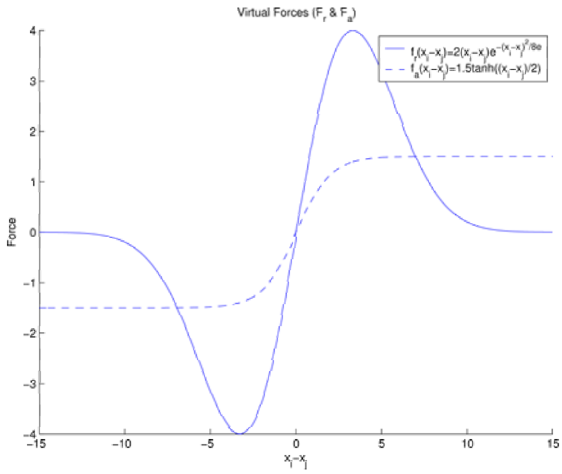

Fig. 1. Virtual repulsive and attractive forces between two agents.

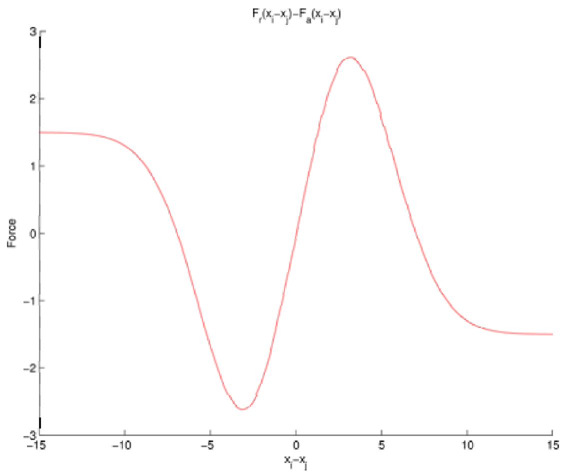

Fig. 2. Combined effect of virtual repulsive and attractive forces.

$\hat{d}_0$. Clearly, if the spacing between two agents (d_{ij}) is greater than $\hat{d}_0$, the attractive force will dominate, and the agents will be dragged closer. If the agents are too close ($d_{ij} < \hat{d}_0$), the repulsive force will dominate, and they will be pushed away from each other. That is, an agent stays balanced from another at a distance around $\hat{d}_0$. Based on this characteristic, we select $d_0 = \hat{d}_0$ in (6).

Remark 2: Note that at an extremely large spacing ($d_{ij} \gg \hat{d}_0$), the combined effect ($f_r - f_a$) of the above virtual forces is a constant, which is not really close to the nature.

With the virtual forces in (8), the decentralized controller is

$$u_i = -m_i k_p\{v_i - \nabla_{x_i} J(x)\} - m_i k_a \sum_{j\in\mathbb{N}_i} \tanh\frac{x_i - x_j}{2} + m_i k_r \sum_{j\in\mathbb{N}_i}(x_i - x_j)\exp(-\frac{\|x_i - x_j\|^2}{c}). \tag{9}$$

To simplify the proving of the velocity convergence, we apply the adjacent matrix in algebraic graph theory [34] to represent the topology of the swarm.

For a swarm of N agents, define the adjacent matrix $A = [a_{ij}]_{N\times N}$ with

$$a_{ij} = \begin{cases} 0, & j = i \text{ or } j \bar{\in} \mathbb{N}_i \\ 1, & j \in \mathbb{N}_i \end{cases} \tag{10}$$

Obviously, $A^T = A$, A is Hermitian, and 0 is an eigenvalue of A. Because of the fact that $f_r(\cdot)$ and $f_a(\cdot)$ are odd functions as defined in (8), we also have

$$\begin{aligned} a_{ij} f_a(x_i - x_j) &= -a_{ji} f_a(x_j - x_i), \\ a_{ij} f_r(x_i - x_j) &= -a_{ji} f_r(x_j - x_i). \end{aligned} \tag{11}$$

Furthermore, define the degree matrix $S_{N\times N} = diag(s_{ii})$, with

$$s_{ii} = \sum_{j=1}^{N} a_{ij}. \tag{12}$$

Clearly, s_{ii} is the number of ith agent's neighbors.

For a swarm with fixed topology, $\mathbb{N}_i$, $A_{N\times N}$ and $S_{N\times N}$ are time-invariant; but for dynamic topology, they are all time-variant functions. We assume the topological graph of the swarm is strongly connected. We will show that all agents' velocities can converge to a common value with bounded errors no matter whether the swarm topology is fixed or dynamic.

Remark 3: Note that, because the definition of the neighborhood is based on the sensing range, it is clear to see that the topological graph represented by $A_{N\times N}$ refers to the sensing graph, rather than commonly used communication graph in literature [28][29] [30][31]. The advantage of this configuration is that by the controller in this paper, the agents do not need to communicate their states with each other and the communication module is not necessary. Subsequently all the assumption and issues about communication delay are relieved.

With the adjacent matrix $A_{N\times N}$ defined in (10), the controller in (9) can be written as

$$u_i = -m_i k_p\{v_i - \nabla_{x_i} J(x)\} - m_i k_a \sum_{j=1}^{N} a_{ij} \tanh\frac{x_i - x_j}{2} + m_i k_r \sum_{j=1}^{N} a_{ij}(x_i - x_j)\exp(-\frac{\|x_i - x_j\|^2}{c}). \tag{13}$$

Thus, (3) becomes

$$\begin{aligned} \dot{\bar{v}} = \frac{1}{N}\sum_{i=1}^{N}\dot{v}_i = &-\frac{1}{N}\sum_{i=1}^{N} k_p v_i + \frac{1}{N}\sum_{i=1}^{N} k_p \nabla_{x_i} J(x) \\ &+ \frac{1}{N}\sum_{i=1}^{N}\sum_{l=1}^{N} k_r a_{il}(x_i - x_l)\exp(-\frac{\|x_i - x_l\|^2}{c}) \\ &- \frac{1}{N}\sum_{i=1}^{N}\sum_{l=1}^{N} k_a a_{il} \tanh(\frac{x_i - x_l}{2}). \end{aligned} \tag{14}$$

From (11) it is not hard to see

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{l=1}^{N} k_r a_{il}(x_i - x_l)\exp(-\frac{\|x_i - x_l\|^2}{c}) = 0, \\ &\sum_{i=1}^{N}\sum_{l=1}^{N} k_a a_{il} \tanh(\frac{x_i - x_l}{2}) = 0. \end{aligned}$$

Then

$$\dot{\bar{v}} = -\frac{1}{N}\sum_{i=1}^{N} k_p v_i + \frac{1}{N}\sum_{i=1}^{N} k_p \nabla_{x_i} J(x). \quad (15)$$

Substituting it into (5), we have

$$\begin{aligned} \dot{e}_{v_i} =& -k_p v_i + \frac{1}{N}\sum_{j=1}^{N} k_p v_j + k_p \nabla_{x_i} J(x) \\ & - \frac{1}{N}\sum_{j=1}^{N} k_p \nabla_{x_j} J(x) - \sum_{j=1}^{N} k_a a_{ij} \tanh(\frac{x_i - x_j}{2}) \\ & + \sum_{j=1}^{N} k_r a_{ij}(x_i - x_j)\exp(-\frac{\|x_i - x_j\|^2}{c}) \end{aligned} \quad (16)$$

Because

$$k_p v_i - \frac{1}{N}\sum_{j=1}^{N} k_p v_j = k_p v_i - k_p \bar{v} = k_p e_{v_i},$$

then

$$\begin{aligned} \dot{e}_{v_i} =& -k_p e_{v_i} + k_p \nabla_{x_i} J(x) - \frac{1}{N}\sum_{j=1}^{N} k_p \nabla_{x_j} J(x) \\ & - \sum_{j=1}^{N} k_a a_{ij} \tanh(\frac{x_i - x_j}{2}) \\ & + \sum_{j=1}^{N} k_r a_{ij}(x_i - x_j)\exp(-\frac{\|x_i - x_j\|^2}{c}). \end{aligned} \quad (17)$$

For simplicity, we write the error dynamics as

$$\dot{e}_{v_i} = -k_p e_{v_i} + f_i + g_i + h_i \quad (18)$$

with

$$f_i = k_p \nabla_{x_i} J(x) - \frac{1}{N}\sum_{j=1}^{N} k_p \nabla_{x_j} J(x), \quad (19)$$

$$g_i = -\sum_{j=1}^{N} k_a a_{ij} \tanh(\frac{x_i - x_j}{2}), \quad (20)$$

$$h_i = \sum_{j=1}^{N} k_r a_{ij}(x_i - x_j)\exp(-\frac{\|x_i - x_j\|^2}{c}). \quad (21)$$

Since

$$\begin{aligned} \|f_i\| &= \|k_p \nabla_{x_i} J(x) - \frac{1}{N}\sum_{j=1}^{N} k_p \nabla_{x_j} J(x)\| \\ &= \|\frac{1}{N} k_p \sum_{j=1}^{N} \{\nabla_{x_i} J(x) - \nabla_{x_j} J(x)\}\|, \end{aligned}$$

for any environment whose potential profile has finite bounded slopes, both $\nabla_{x_i} J(x)$ and $\nabla_{x_j} J(x)$ are bounded, then $\nabla_{x_i} J(x) - \nabla_{x_j} J(x)$ is also bounded, so $\|f_i\|$ is bounded, denoted as

$$\|f_i\| \le \alpha_i \quad (22)$$

In particular, if $J(x)$ is a linear profile, i.e., $J(x) = \sum_{i=1}^{N} R_i x_i$ where R_i's are given row vectors, then $\alpha_i = \|f_i\| = \|k_p(R_i - \bar{R})\|$, in which $\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$.

From the bounded feature of $\tanh(\cdot)$ function, it is obvious that

$$\|g_i\| \le \|\sum_{j=1}^{N} k_a a_{ij}\| = s_{ii} k_a \le (N-1)k_a. \quad (23)$$

From the fact that $r\exp(-\frac{r^2}{c}) \leqslant \sqrt{c/(2e)}$, it is not hard to have

$$\begin{aligned} \|h_i\| &\le \|\sum_{j=1}^{N} k_r a_{ij}\sqrt{c/(2e)}\| \\ &= s_{ii} k_r \sqrt{c/(2e)} \le (N-1)k_r\sqrt{c/(2e)}. \end{aligned} \quad (24)$$

Thus

$$\begin{aligned} &\|f_i + g_i + h_i\| \le \|f_i\| + \|g_i\| + \|h_i\| \\ &\quad \le \alpha_i + (N-1)k_a + (N-1)k_r\sqrt{c/(2e)} \triangleq \beta_i, \end{aligned} \quad (25)$$

in which β_i is a positive number.

Therefore, we have

$$\dot{e}_{v_i} \le -k_p e_{v_i} + \beta_i \cdot \mathbf{1}_{n\times 1}, \quad (26)$$

where $\mathbf{1}_{n\times 1}$ is a column vector with all 1's.

By comparison principle, we know $e_{v_i}(t)$ is bounded on every dimension, thus $e_{v_i}(t)$ is bounded, which means all agents' velocities converge to a common value with bounded errors.

And if the environment has an identical effect on each agent, that is, the upper bound α_i's in (22) are the same for each agent, then we have all β_i's in (25) to be same.

Remark 4: Note that the preceding proof holds without any specific condition about the swarm topology. It means that the decentralized controller in (9) can achieve the collective behavior of the swarm, no matter its topology is fixed or dynamic.

Remark 5: Note that, when d_{xj} is zero, i.e., two agents come together, both repulsive and attractive forces in (8) are zeroes. From the controller in (9) we know that the virtual forces can affect the spacing between two agents. Thus, if the agents move in a same environment and both virtual forces are zeroes, the spacing between two agents will keep unchanged. This means that some agents may become overlapped during the swarm's motion.

Furthermore, we discuss the time for all agents' velocities to converge to a common value with certain errors. Without loss of generality, we consider the one dimensional case, i.e., v_i, $e_{v_i} \in \mathbb{R}$. By comparison principle, from (26) we have

$$e_{v_i}(t) \le e_{v_i}(0)\cdot e^{-k_p t} + \frac{\beta_i}{k_p}[1 - e^{-k_p t}], \quad (27)$$

in which $e_{v_i}(0)$ is the difference between the ith agent's velocity and the average velocity of all agents at initial stage. From (27) it is clear to see that for each agent, the steady error is bounded by

$$\lim_{t\to\infty} e_{v_i}(t) \le \frac{\beta_i}{k_p}$$

For simplicity we denote the right hand side of (27) as

$$\eta_i(t) = e_{v_i}(0)\cdot e^{-k_p t} + \frac{\beta_i}{k_p}[1 - e^{-k_p t}], \quad (28)$$

then

$$\dot{\eta}_i(t) = [-k_p e_{v_i}(0) + \beta_i]e^{-k_p t}. \quad (29)$$

If $e_{v_i}(0) > \frac{\beta_i}{k_p}$, i.e., $-k_p e_{v_i}(0) + \beta_i < 0$, then $\dot{\eta}_i(t) < 0$, $\eta_i(t)$ is strictly decreasing. Let t_{η_i} denote the time when η_i come to a positive number ε which is bigger but arbitrarily close to $\frac{\beta_i}{k_p}$. Then

$$\eta_i(t_{\eta_i}) = e_{v_i}(0) \cdot e^{-k_p t_{\eta_i}} + \frac{\beta_i}{k_p}[1 - e^{-k_p t_{\eta_i}}] = \varepsilon,$$

that is,

$$[e_{v_i}(0) - \frac{\beta_i}{k_p}]e^{-k_p t_{\eta_i}} = \varepsilon - \frac{\beta_i}{k_p},$$

so we have

$$t_{\eta_i} = \frac{1}{k_p} \ln \frac{e_{v_i(0)} - \beta_i/k_p}{\varepsilon - \beta_i/k_p}. \tag{30}$$

Since $e_{v_i}(t)$ is upper bounded by $\eta_i(t)$, it is clear that by the time $t = t_{\eta_i}$, it will be $e_{v_i} \leq \varepsilon$. This means that the velocity of agent i converges to the average value of all agents' velocities with error ε by the time t_{η_i}. As we can see from (30), the convergence time will depend on $e_{v_i}(0)$. If initially $e_{v_i}(0)$'s are big, that is, the velocity of each agent is much more variant from each other, the group needs more time to converge to the common velocity.

If $e_{v_i}(0) < \frac{\beta_i}{k_p}$, then $\eta_i(t)$ is strictly increasing. And from (28), we have $\lim_{t\to\infty} \eta_i(t) = \beta_i/k_p$, so $\eta_i(t)$ is always smaller than β_i/k_p. Since $e_{v_i}(t)$ is upper bounded by $\eta_i(t)$, therefore, $e_{v_i}(t)$ is also always smaller than β_i/k_p, which means the convergence of the velocities of all agents is always bounded. If $e_{v_i}(0) = \frac{\beta_i}{k_p}$, this is the trivial case in (27), in which $e_{v_i}(t)$ will always be not bigger than β_i/k_p.

In particular, if $e_{v_i}(0) = 0$, then $e_{v_i}(t) \leq \frac{\beta_i}{k_p}[1 - e^{-k_p t}]$. For arbitrary small number $\varepsilon < \beta_i/k_p$,

$$\frac{\beta_i}{k_p}[1 - e^{-k_p t}] < \varepsilon \Rightarrow t < \frac{1}{k_p} \ln \frac{\beta_i}{\beta_i - k_p \varepsilon}. \tag{31}$$

Then, the time for e_{v_i} to become smaller than ε is less than $\frac{1}{k_p} \ln \frac{\beta_i}{\beta_i - k_p \varepsilon}$.

IV. SIMULATIONS

In this section, a few sets of simulation results are presented to demonstrate the feasibility of the proposed controller in previous section.

All swarm members are assumed to know their own masses, and have identical sensing ranges. Their initial positions and velocities are randomly given. We use the virtual forces as in (8), and the design constants are $k_p = 3.5$, $k_r = 2.0$, $c = 8e$, $k_a = 1.5$. We select the neighborhood defining distance $d_0 = \hat{d}_0 = 6.98$. In the following figures, the stars and circles represent agents' initial and final positions in the simulation duration, respectively.

Fig. 3 shows the agents' positions and velocities when the swarm ($N = 8$) moves in an environment with linear potential profile. Select all R_i's to be -1.0. It is clear to see that, as expected, all agents' velocities approach to a same value as time evolves.

Fig. 4–6 show the case when the swarm ($N = 50$) moves in a 2D environment with linear potential profile. Assume for all agents $R_i = [-0.5, -0.5]$. All agents' initial positions are randomly located in a 3×3 square centered at zero, thus, initially they are neighbors of each other. As time evolves, the agents' speeds converge to a common value. Fig. 4 shows how the swarm members' positions evolve with time, and Fig. 5 shows their trajectories on $x-y$ plane. The convergence of all velocities is shown in Fig. 6.

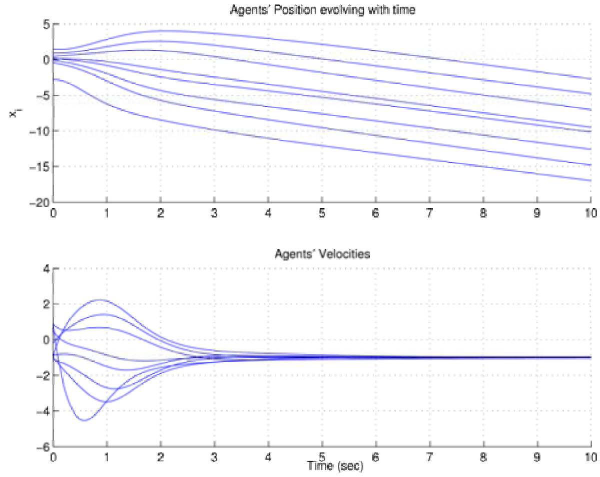

Fig. 3. Agents' positions and velocities when moving in a linear 1D environment ($N = 8$).

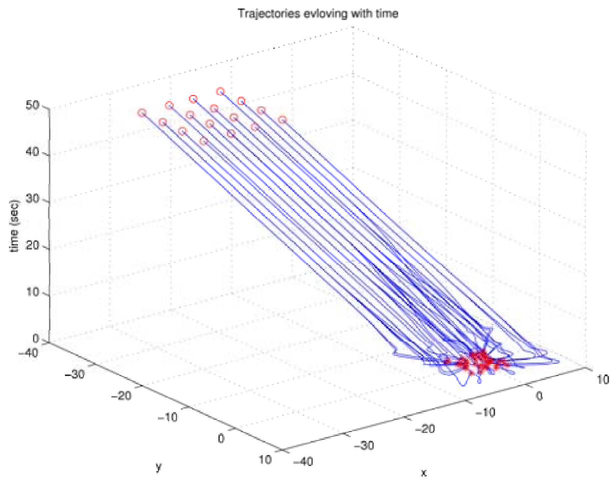

Fig. 4. Agents' positions evolving with time when moving in a linear 2D environment ($N = 50$).

Fig. 7 show the swarm's motion in a 2D environment with sinusoid ($N = 10$) wave profile. All agents' initial positions are randomly located in a 3×3 square centered at zero. It is clear to see that all swarm members can eventually move together to follow the sinusoid "wave" of the environment, as a group of fish schooling along the ocean currents.

Furthermore, we simulate swarms flocking in an environment with certain agents being lost. In the following Fig. 8 – 10, ten agents flock in an environment with linear profile, and two agents are lost at a certain time. Fig. 9 shows the patterns of swarms' spatial topology before agents being lost. Fig. 10 shows the pattern after agent 3 and 6 being lost. From the pattern change, we can see that the lost agents will not

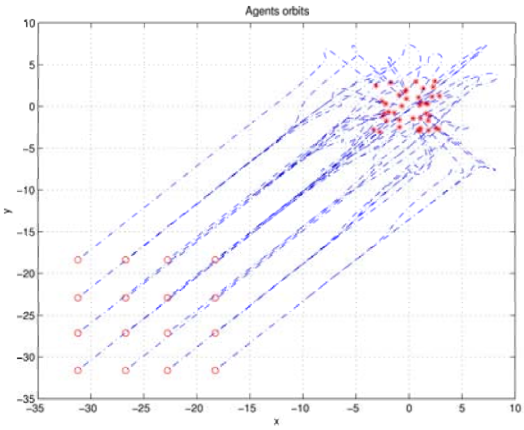

Fig. 5. Agents' trajectories in x-y plane when moving in a linear 2D environment ($N = 50$).

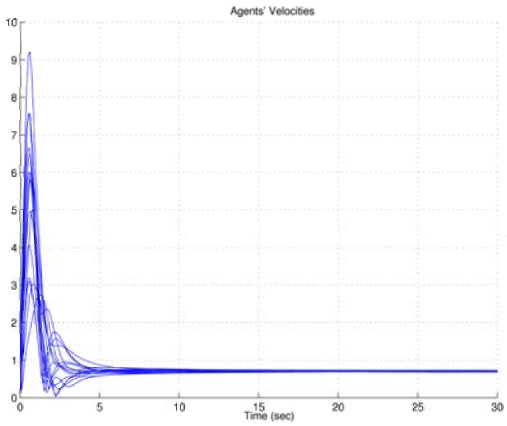

Fig. 6. Agents' velocities when moving in a linear 2D environment ($N = 50$).

affect those swarm-mates who are far away from them, but for those who are not even their nearest neighbors but close enough, the loss has certain effects. However, as we can see in the simulation, in spite of the loss of agents, the controller is still robust enough to achieve the convergence of velocities .

Note that, in the above simulation results, the spacings between the agents are less than the balance distance $\hat{d}_0$. This is reasonable since the balance distance $\hat{d}_0$ is defined for only two agents. As the size of the swarm increases, there will be more virtual forces played upon every agent, subsequently the ultimate balance will occur at a distance less than $\hat{d}_0$.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we investigate the mechanism behind the phenomena of a swarm of mobile agents flocking in certain environments. Such phenomena happen in examples such as schooling fish, flocking birds, and herding beasts. We study these phenomena and model the swarm members individually. Inspired by some biological observations and hypothesis, we propose a general decentralized controller for each agent to make the swarm members flock together. We present a new

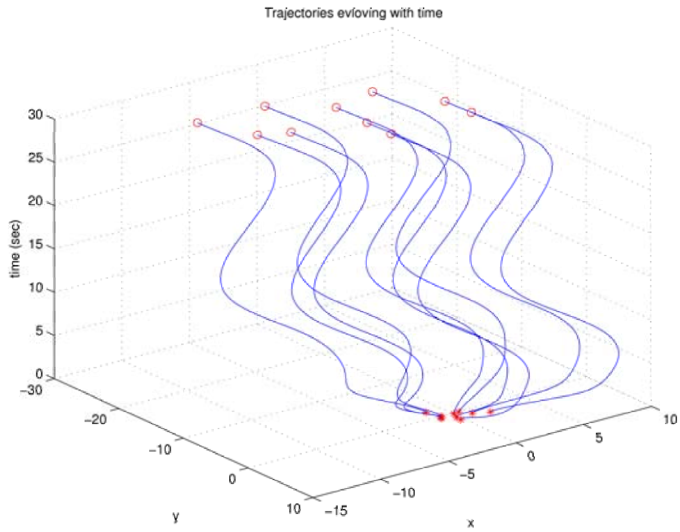

Fig. 7. Agents' Motion is a 2D sinusoid wave environment ($N = 10$).

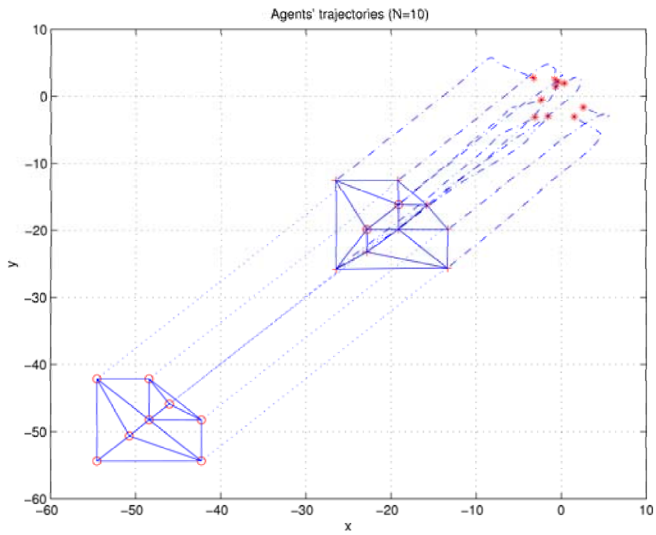

Fig. 8. Agents' trajectories (N=10) with two agents being lost.

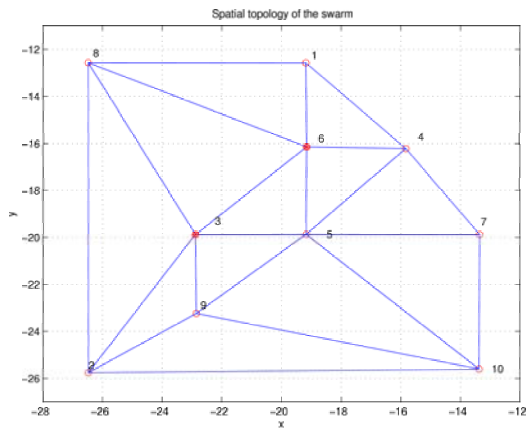

Fig. 9. Swarm's spatial topology in Fig. 8 before agents being lost.

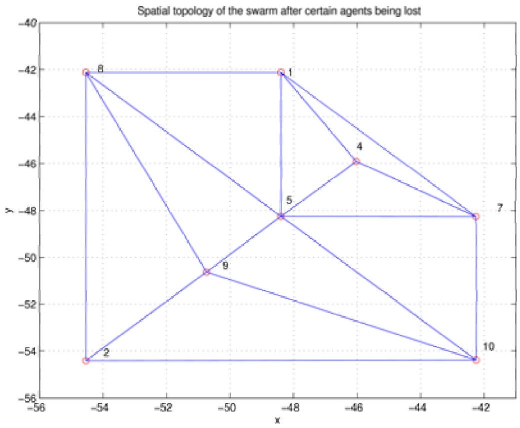

Fig. 10. Swarm's spatial topology in Fig. 8 after agent 3 and 6 being lost.

set of virtual force functions, and show the proposed controller can lead all agents' velocities to converge a common value with bounded errors. The contributions of this paper are two folds. First, the controller needs only local and relative information to achieve the collective behavior of the swarm, which makes the global position sensing device and communication module not needed. Second, a new set of functions is presented for the mutual interaction force among swarm members.

As discussed in Remark 2 and 5, the proposed virtual force functions have a non-zero combined effect when two agents are extremely far away from each other, and certain agents may become overlapped during their motions. These are not exactly matched with the reality. Finding a more appropriate function will be our next research target. The open problem about the constant spacing, which is independent of swarm size, also needs to be investigated thoroughly. Moreover, in order to have a better solution for engineering applications, we need to consider the disturbances upon agents, the sensing noises, and fluctuations of the environment. All these issues can be our future research topics.

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