

# Research Advances in Vehicle Lateral Motion Monitoring and Control

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*Abstract*- This paper presents a review of recent developments and trends in vehicle lateral (steering) monitoring and control research. By scanning the highly multidisciplinary and quickly developing research in this area, this paper explains the initiatives and prime techniques applied with an emphasis on steering observer and controller design. Specially, some important vehicle steering controller design problems are carefully discussed as well as current main solving approaches. Vehicle motion sensory (real sensor), observer (virtual sensor) and their relationships are also carefully studied. Besides, current developments of steer-by-wire systems are briefly examined.

*Index Terms*—Vehicle lateral motion, vehicle sensory, steering controller design, steer-by-wire.

## 1. INTRODUCTION

Increasing traffic congestion and accidents inspired the concepts of Automated Highway Systems (AHS) and Intelligent Vehicle Intuitive (IVI) more than twenty years ago. Among a variety of techniques that had been introduced, the concept of Advanced Vehicle Control Systems (AVCS) gains significant interests world-widely [1]-[5].

Advanced vehicle control systems help drivers by taking control of the steering, brakes and/or throttle to maneuver the vehicle in a safe state. The related technologies include smart cruise control, collision avoidance systems, and platooning control, etc. On the aspects of single vehicle motion control, associated research covers: lateral control, longitudinal control and their combinations.

The main task of longitudinal control is vehicle following/ tracking. It requires an appropriate headway to be maintained between the leading vehicle and the controlled vehicle to avoid collision. The lateral control usually refers to vehicle steering control. Its prime task is path (lane) following, or more plainly, to keep the vehicle on the road. More specifically, research in this field mainly addresses on the following topics:

1) vehicle lateral motion modeling, which is tightly related to tire/road friction modeling;

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2) vehicle lateral motion related sensory, which includes vehicle position sensory, lane detection rotation, and vehicle movement measurement etc.;

4) vehicle steering actuator implementation. One hot topic in this area is steer-by-wire;

5) vehicle lateral motion monitoring;

6) vehicle lateral motion control;

7) vehicle lateral motion and driver assistance.

This paper sequentially looks into the above areas of vehicle lateral motion control associate researches, except the last one. Since discussion on driver assistance requires a dedicated paper, the related issues will not be separately examined here.

Because of vehicle model variety and simulation/testing parameters difference, this paper does not intend to make a thorough comparison among numerous different observers/controllers that had been proposed, since it is too difficult.

The goal of this paper is to provide a state-of-art survey of research works in lateral vehicle monitoring and control, and to offer a multidisciplinary perspective for researchers who are involved in this field. Although it is impossible to cover the large number of publications in this area, the key research findings and trends are included. Especially, the focus is on recent literature, since excellent reviews already exist in the relatively long history of the associated studies [5], [12], [15], [44], [68], [83], [138].

The other sections are organized as follows. In section 2, an object model for collaborative systems (OMCS) and a multimedia co-authoring system (MCAS) are introduced briefly and the conflict resolution structure with roles used in our system MCAS is described; in section 3, conflict resolution at the management level is discussed and the regulations for role management used in the interface design is demonstrated by Petri nets; in section 4, the conflict resolution at the document level is illustrated and the regulations of editing operations used in the design of a manager object are depicted; in section 5, some implementation issues for the method with roles in the MCAS system are outlined; and in the last section, the paper is concluded that a method with roles can help to resolve conflicts in collaborative systems and suggestions are provided on how to introduce roles into designing collaborative systems.

## 2. VEHICLE LATERAL MOTION MODELING

Generally, there are two types of wheeled vehicles: single track vehicles and track-trailers. Track-trailer systems consist of a steering tractor and one (sometimes more than one) passive trailer(s) linked with rigid free

joints. Single track vehicles usually refer to passenger cars or car-like vehicles/robots which can be viewed as a single steering tractor. From the viewpoint of steering, single track vehicles can be further classified into two types: front steering vehicles (2WS) in which only the two front tires can be steered, and full steering vehicles (4WS) of which the front and rear tires can be steered independently. Because to discussion track-trailers steering control requires a dedicated paper due to its broad range, this paper addresses on single track vehicle in the rest of this paper.

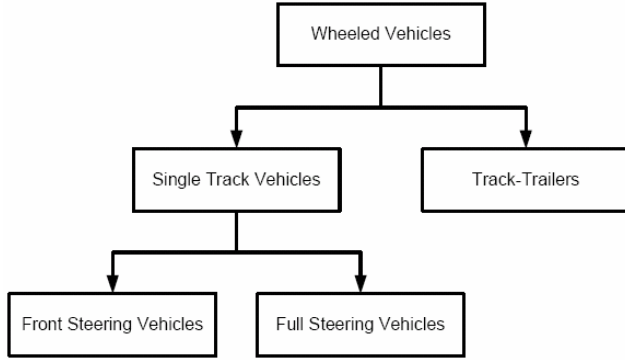


Fig.1. Classification of vehicles based on lateral motion model.

## 2.1 Bicycle Model

Lateral vehicle dynamics has been studied since the 1950s [6]-[7]. In 1956, Segel presented a vehicle model with three degrees of freedom in order to describe lateral movements including roll and yaw. If roll movement is neglected, a simple model known as "bicycle model" is obtained. This model is widely used for studies of lateral vehicle dynamics (yaw and sideslip) now. The following discussions will be primly carried out based on this model.

Suppose the vehicle is moving on a flat surface. By lumping the four wheels into one virtual wheel in the centerline of the vehicle, we can have front steering and full steering models as shown in Fig.2(a) and Fig.2(b) respectively [8]-[10].

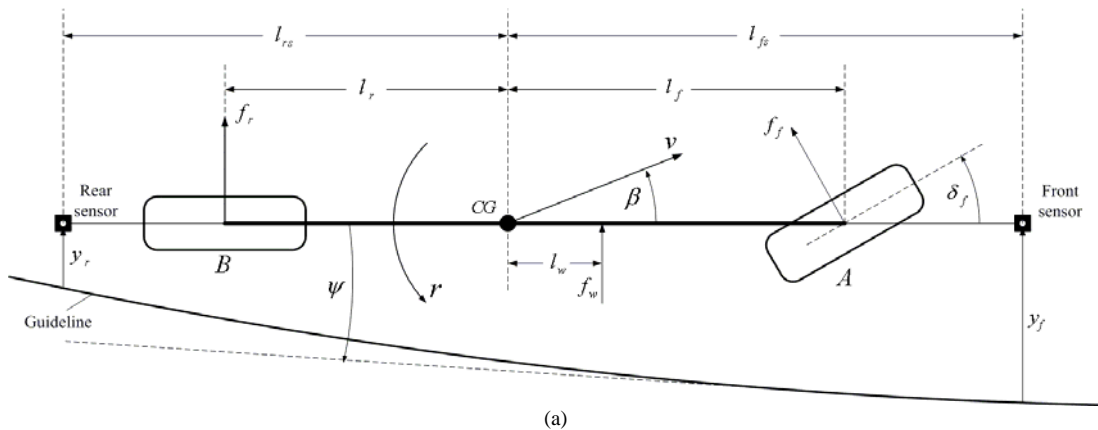
Here, reference point  $CG$  is chosen to represent the center of gravity for vehicle body, where vehicle velocity

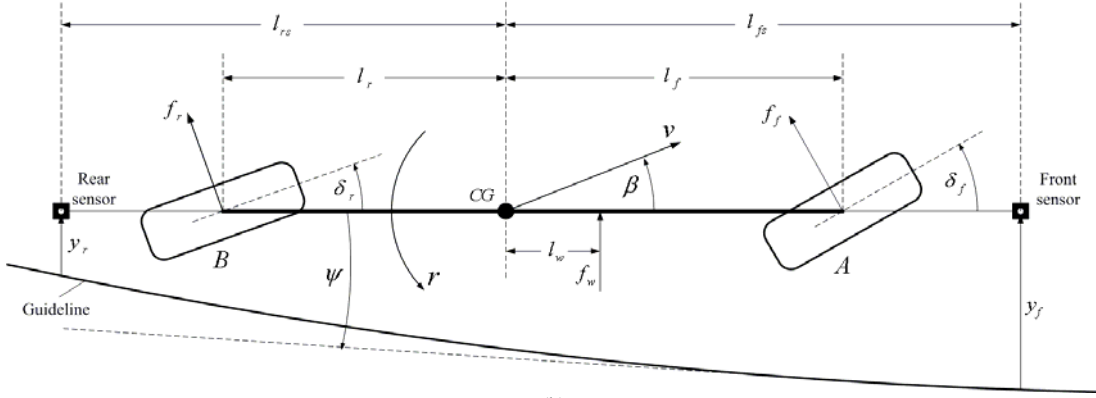
$v$  is defined. Symbol  $A$  and  $B$  denote the positions of front and rear tire/road interfaces respectively. Heading angle  $\psi$  is the angle from the guideline to the longitudinal axis of vehicle body  $AB$ . Slide-slip angle  $\beta$  is the angle from the longitudinal axis of vehicle body to the direction of the vehicle velocity.  $\delta_f$  is the front tire steering angle, and  $\delta_r$  is the rear tire steering angle. Yaw rate is denoted as  $r$ .  $f_f$  and  $f_r$  are the front and rear tire forces which are perpendicular to the directions of tire movements, respectively.  $f_w$  is the wind force acting on the aerodynamic center of the side surface and  $l_w$  denotes the distance between  $CG$  and aero-dynamical center of the side surface.  $l_{fs}$  and  $l_{rs}$  denotes the distances from the front and rear sensor "looking at" points to  $CG$ , respectively.  $y_f$  and  $y_r$  represent the displacements from the front and rear "looking at" points to the guideline. Other variables are given in Table I, in which the typical values are set for a city bus O 305 based on IFAC benchmark example [9]. Here,  $c_f$  and  $c_r$  denote the cornering stiffness of front and rear tires respectively, which will be introduced in the following Eqs.(7)-(8).

Table 1: Parameters and their typical values [9]

Symbols	Typical values
Mass of the vehicle $m$	[9950, 16000]kg
Inertia moment around z-axis $I_z$	[10.85, 21.7]Ns/rad
Distance from $A$ and $CG$ $l_f$	3.67m
Distance from $B$ and $CG$ $l_r$	1.93m
$l_{fs}$	6.12m
Stiffness coefficients of front tire $c_f$	198000N/rad
Stiffness coefficients of rear tire $c_r$	470000N/rad

Assuming that vehicle has a constant velocity, front steering model with nonlinear tire force characteristics can be described by the differential equations





(b)  
Fig.2. "Bicycle" steering model: front steering (a) and full steering (b).

$$\frac{d}{dt} \begin{pmatrix} \beta \\ r \end{pmatrix} = \begin{pmatrix} \frac{f_f + f_r}{mv} - r \\ \frac{l_f f_f - l_r f_r}{I_z} \cos \beta \end{pmatrix} \quad (1)$$

Using the famous "magic formula" of tire/road friction given in [11]-[12], we have

$$f_f = D_f \sin \left\{ C_f \tan^{-1} \left( B_f [1 - E_f] \alpha_f + E_f \tan^{-1} (B_f \alpha_f) \right) \right\} \quad (2)$$

$$f_r = D_r \sin \left\{ C_r \tan^{-1} \left( B_r [1 - E_r] \alpha_r + E_r \tan^{-1} (B_r \alpha_r) \right) \right\} \quad (3)$$

$$\begin{cases} \alpha_f = \beta + \tan^{-1} \left( \frac{l_f}{v} \cdot r \cos \beta \right) - \delta_f \\ \alpha_r = \beta - \tan^{-1} \left( \frac{l_f}{v} \cdot r \cos \beta \right) \end{cases} \quad (4)$$

where  $\alpha_f$  is the slip angle of front tires,  $\alpha_r$  is the slip angle of rear tires. The coefficients  $B_j$ ,  $C_j$ ,  $D_j$  and  $E_j$  ( $j = f, r$ ) in the models can be calculated in practice.

In [13]-[14], the bifurcation phenomena for the above model (1)-(4) are analyzed. The vehicle unstabilization was shown to be caused by a saddle-node bifurcation which depends heavily on the rear tire side force saturation. By approximating the nonlinearities with

$$\cos \beta \cong 1, \quad \alpha_f \cong \beta + \frac{l_f}{v} \cdot r + \delta_f, \quad \alpha_r \cong \beta - \frac{l_f}{v} \cdot r \quad (5)$$

they get the Jacobian matrix of  $\frac{d}{dt} \begin{pmatrix} \beta \\ r \end{pmatrix} = F(\beta, r, \delta_f)$

at equilibrium point  $\chi_0$  as

$$A_{\chi_0} = \begin{bmatrix} -\frac{c_f^* + c_r^*}{mv} & -1 - \frac{l_f c_f^* - l_r c_r^*}{mv^2} \\ -\frac{l_f c_f^* - l_r c_r^*}{I_z} & -\frac{l_f^2 c_f^* + l_r^2 c_r^*}{I_z v} \end{bmatrix} \quad (6)$$

where  $c_f^*$  and  $c_r^*$  are the tangents to slopes of front and rear side force characteristics at equilibrium point  $\chi_0$  respectively. The bifurcation situation around point  $\chi_0$

has been checked.

If cornering stiffness  $c_f^*$  and  $c_r^*$  is taken to be constant, we can write the linear dynamic model for front steering vehicle as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{y}_f \\ \dot{y}_r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ v & l_{fs} & v & 0 & 0 \\ -v & -l_{rs} & v & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \\ y_f \\ y_r \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & d_1 \\ b_{21} & 0 & d_2 \\ 0 & -v & 0 \\ 0 & -vl_{fs} & 0 \\ 0 & vl_{rs} & 0 \end{bmatrix} \begin{bmatrix} \delta_f \\ \rho_{ref} \\ f_w \end{bmatrix} \quad (7)$$

Similarly, the linear dynamic model of full steering vehicle is written as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{y}_f \\ \dot{y}_r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ v & l_{fs} & v & 0 & 0 \\ -v & -l_{rs} & v & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \\ y_f \\ y_r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & 0 & d_1 \\ b_{21} & b_{22} & 0 & d_2 \\ 0 & 0 & -v & 0 \\ 0 & 0 & -vl_{fs} & 0 \\ 0 & 0 & vl_{rs} & 0 \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ \rho_{ref} \\ f_w \end{bmatrix} \quad (8)$$

where

$$a_{11} = -(c_r + c_f) / \tilde{m}v, \quad a_{12} = -1 + (c_r l_r - c_f l_f) / \tilde{m}v^2$$

$$a_{21} = (c_r l_r - c_f l_f) / \tilde{I}_z, \quad a_{22} = -(c_r l_r^2 + c_f l_f^2) / \tilde{I}_z v$$

$$b_{11} = c_f / \tilde{m}v, \quad b_{12} = c_r / \tilde{m}v, \quad b_{21} = c_f l_f / \tilde{I}_z$$

$$b_{22} = -c_r l_r / \tilde{I}_z, \quad d_1 = 1 / mv, \quad d_2 = l_w / I_z$$

Here  $\tilde{m} = m / \mu$  and  $\tilde{I}_z = I_z / \mu$  are the normalized mass and inertia respectively, in which  $\mu$  is common road adhesion factor.  $\rho_{ref}$  is the curvature of the guideline. The contribution of  $\rho_{ref}$  to  $y_f$  or  $y_r$  is sometimes neglected, since it is small.

It should be pointed out that cornering stiffness varies with several factors. One well known fact is that it increases with tire pressure. When the car turns, the mass transfer onto the external wheels increases tire pressure, which can lead to notable variations in cornering stiffness. Fortunately, Stephant, Charara and Meizel showed in [15] that such variations are normally less than 10% and still tolerant for most robust steering controllers, see Fig.3.

Full steering vehicles significantly outperform front

steering vehicles in handling and stability [16]-[17]. Usually, when the vehicle enters the curved path, the rear wheel will first steer in the opposite direction to the front wheel in order to generate sufficient yaw motion to follow the desired yaw rate. After that, the rear wheel steers will synchronize with the front wheel to keep the yaw rate with desired value and also control the lateral motion for path tracking. Since most controller design methods can be applied to both situations without tedious modifications, the difference between front steering vehicles and full steering vehicles will not be emphasized in the rest of this paper.

Since "bicycle model" captures the prime characteristics of vehicle steering movement and yields a relatively simple linear model for analyzing, it is now widely used in steering controller design.

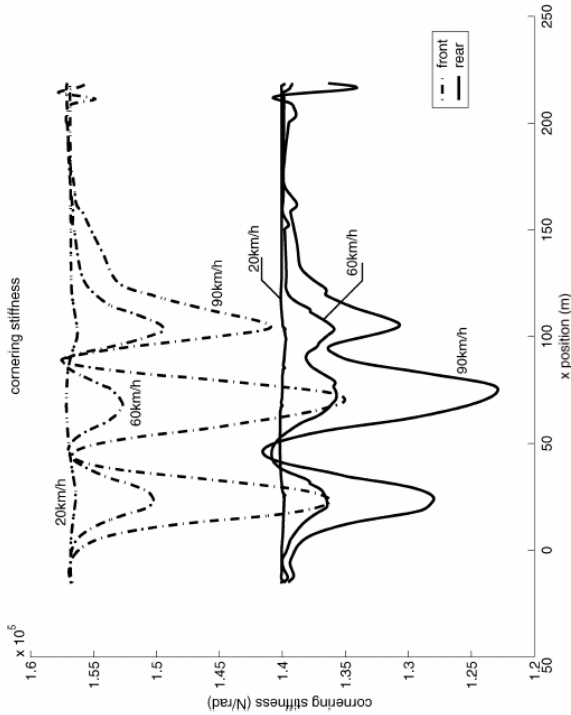


Fig.3. Variation of rear and front cornering stiffness at speed 20, 60, and 90 km/h [46]

## 2.2 Other Vehicle Lateral Motion Models

Besides the "bicycle model", there were some other dynamic models that had been proposed and analyzed. Usually, they differ from "bicycle model" in two aspects:

- 1) apply more complex tire/road friction models other than the "magic formula" (2)-(3) [19]-[22];
- 2) set up integrated vehicle motion model which consider longitudinal and lateral motion simultaneously [23]- [25].

For instance, the lateral friction force is calculated as

below in [19]

$$F_{yf} = \left[ C_f'' \frac{C_f'^2}{\mu} \frac{C_f'^3}{\mu^2} \right] \cdot \left[ \left| \tan \alpha_f \right| \frac{-\left| \tan \alpha_f \right|^2}{3F_z} \frac{-\left| \tan \alpha_f \right|^3}{27F_z^2} \right]^T \quad (9)$$

It has been proven in several reports, i.e. [20], that such nonlinear proportional models can provide more accurate descriptions for the lateral tire/road friction phenomena and are still easy to identification.

For the second cases, these models usually address the effect of vehicle longitudinal velocity variation on the lateral motion. Many of them directly build up differential equation sets of longitudinal and lateral velocity, and do not explicitly consider vehicle slide-slip angle  $\beta$ .

For example, the vehicle dynamic model proposed in [23] tried to incorporate vehicle aerodynamics into lateral motion model. It was represented as

$$\begin{cases} \dot{v}_x = \frac{1}{m} [T + mv_y r - mfg + c_f \frac{v_y + l_f r}{v_x} \delta_f + v_x^2 (fk_1 - k_2)] \\ \dot{v}_y = \frac{1}{m} [(c_f + T) \delta_f - (c_f + c_r) \frac{v_y}{v_x} - (l_f c_f - l_r c_r + mv_x^2) \frac{r}{v_x}] \\ \dot{r} = \frac{1}{I_z} [(l_f c_f + T) \delta_f - (l_f c_f - l_r c_r) \frac{v_y}{v_x} - (l_r^2 c_r + l_f^2 c_f) \frac{r}{v_x}] \end{cases} \quad (10)$$

where  $v_x$  and  $v_y$  are vehicle's longitudinal velocity and lateral velocity in vehicle coordinate respectively.  $T$  is the traction or braking force.  $f$  is the rotation coefficients.  $k_1$  and  $k_2$  are lift and drag parameters from aerodynamics respectively.

Integrated vehicle motion models are capable to approximate real vehicle dynamics more accurately. There is considerable theoretical and experimental research on developing vehicle models of different levels of complexity [5], [21]. Thorough discussions on this issue requirement a dedicated publication, and are thus omitted in this paper.

## 2.3 Vehicle Lateral Motion Objectives

The design specifications of steering controller are primarily given in terms of maximal displacement from the guideline; while the main constraints are actuator saturation which can be described by maximal steering angle and steering angle rate. For instance, the benchmark problem mentioned in [9] mainly requires

- 1) the steering angle is limited as  $\|\delta_f\| \leq 40 \text{ deg}$  (11)

- 2) the steering angle rate is limited as  $\|\dot{\delta}_f\| \leq 28 \text{ deg/s}$  (12)

- 3) the displacement from the guideline  $y_f$  and  $y_r$  must not exceed 0.15m in transient state and

0.02m in steady state;

- 4) the lateral acceleration must not exceed 2m/s for passengers comfort. The ultimate limit is 4m/s.

In some other literals, the safety driving requirements are expressed as an inequality set on vehicle positions in world coordinates, which can be written as

$$\begin{cases} x_{lowerbound}(t) \leq x(t) \leq x_{upperbound}(t) \\ y_{lowerbound}(t) \leq y(t) \leq y_{upperbound}(t) \\ \psi_{lowerbound}(t) \leq \psi(t) \leq \psi_{upperbound}(t) \end{cases} \quad (13)$$

where  $(x, y)$  denote vehicle center gravity's position in world coordinates. And normally, the following mapping function (14) is used to describe the relationship between vehicle velocities denoted in orthogonal coordinates and that denoted in world coordinates [21].

$$\begin{cases} \dot{x} = v_x \cos(\psi) - v_y \sin(\psi) \\ \dot{y} = v_x \sin(\psi) + v_y \cos(\psi) \\ \dot{\psi} = r \end{cases} \quad (14)$$

### 3. ADVANCES IN STEERING CONTROL DEVICES

Increasing requirements of safe and comfortable driving have led vehicle manufacturers and suppliers to actively pursue development programs in the so called "by-wire" subsystems. These computer-controlled subsystems include steer-by-wire, brake-by-wire, drive-by-wire and etc., which are connected through in-vehicle computer networks [26]-[30].

A steer-by-wire system replaces the traditional mechanical linkage between the steering wheel and the road wheel actuator (e.g., a rack and pinion steering system) with an electronic connection. Because it removes direct kinematical relationship between the steering and road wheels, it enables advanced control algorithms to help enhance driver steering command.

For instance, Fig.4(b) shows a production model which was modified for full steer-by-wire capability by replacing the steering shaft with a brushless DC servomotor actuator [29]. A rotary position sensor measures the lower steering shaft angle, which is equal to the front wheel steer angle scaled by the steering ratio. An identical sensor attached to the upper steering shaft measures the hand wheel angle. The servomotor actuator specifications are chosen based on the maximum torque and speed necessary to steer the vehicle under typical driving conditions including moderate emergency maneuvers.

The differential equation describing the steering system dynamics is written as follows:

$$J\ddot{\theta} + b\dot{\theta} + F_c \operatorname{sgn} \dot{\theta} + k_a \tau_a = \tau \quad (15)$$

where  $\theta$  is the pinion angle,  $J$  is the total moment of inertia of the system,  $b$  is viscous damping coefficient,  $F_c$

represents Coulomb friction force,  $k_a$  is a scale factor,  $\tau_a$  is the tire self-aligning moment, and  $\tau$  is the controllable actuator torque.

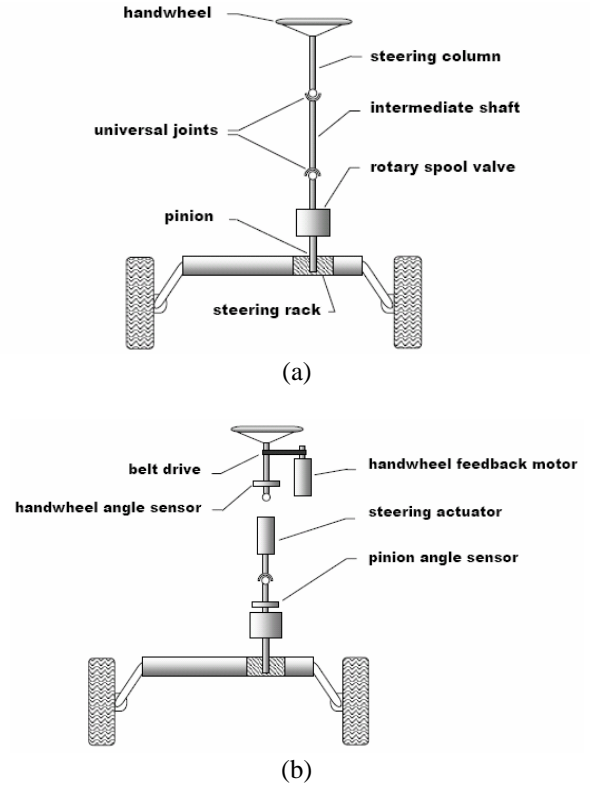


Fig.4. Diagram of steering systems: (a) conventional steering system, (b) steer-by-wire system [29].

In [29], steering torque on average required at the handwheel during normal driving ranges from 0 to 2 Nm, while emergency maneuvers can demand up to 15 Nm of torque. It is reported in [29] that the actuator installed in the test vehicle can provide a maximum steering torque of 17.1 Nm with a maximum steer rate of 700 degrees per second.

The purpose of steer-by-wire controller is to track driver commanded steer angle  $\theta_d$  with minimal error. Normally, the response via steer-by-wire is more accurately and quickly than that in conventional steering system, which improves vehicle stability and provides a basis for fault detection.

Currently, there are four important problems have to be solved before the steer-by-wire system is applied into practical vehicles:

- 1) how to choose fast and stable tracking algorithms or hardware for steer-by-wire systems; see [29]-[33], [34]-[38];
- 2) how to deal with disturbance torque [29], [39]-[40];
- 3) improve the driver usability and manoeuvrability [28], [41]-[43];

- 4) how to implement robust steer-by-wire systems with both fault-tolerant hardware and software [44]-[47].

A steer-by-wire system needs to be robust to the disturbance torque generated by road roughness, and parameter changes caused by tire pressure/temperature and loading variations. For instance, the driver control input in [40] was augmented by the adaptive component (Force comp), which is based on the load experienced by the plant, to reject road feedback (disturbance).

To improve driver input impedance is another hot topic in research of steer-by-wire systems. In [37], the steer-by-wire system is composed of two motors controlled by Electronic control unit (ECU) instead of mechanical linkage. One motor in the steering wheel is to improve the driver's steering feel and the other motor in the steering linkage is to improve the vehicle stability under under-steer and over-steer situations concerning vehicle's velocity.

Some current products provide variable steering ratio for the driver. As mentioned in [43], at low vehicle speeds, the driver prefers a fixed steering ratio, after which the steering ratio increases (becomes increasingly indirect) as the speed increase. A steady-state constant Human-Machine Interaction yaw gain (yaw-rate divided by HMI angle) may be derived from vehicle data or measurements at intermediate and high speeds; see Fig.5. It was also showed in [26]-[30] that by effectively changing steering ratio, the same vehicle can be made to handle differently. Thus, it is possible to maintain consistent handling characteristics under variable operating conditions.

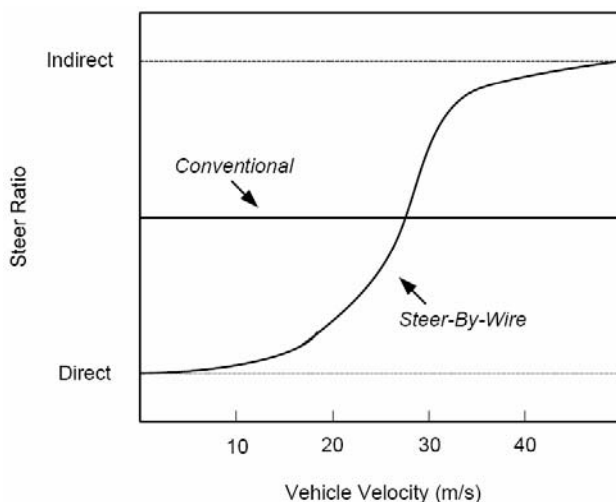


Fig.5. Variable steering ratio for different vehicle velocities.

Fault detection and fault tolerant control are essential parts in steer-by-wire systems. Many topics were presented and studied from the surrounding field in both hardware and software reliability [44]-[47]:

- a) self fault detection and self repair;

- b) online/offline diagnosis of actuators and sensors;
- c) online/offline diagnosis of inner vehicle field bus and communication networks;
- d) online/offline diagnosis testing of control algorithms and modules.

Designers must reduce the chance of faults in safety-critical steer-by-wire systems, and choose an acceptable compromising point between costs and level of risks.

Besides, steer-by-wire technique simplifies assembly and reduces vehicle's mass. For example, in the steer-by-wire system proposed in [29], only the stock hydraulic power assist unit and rack-pinion mechanism is retained. This naturally allows flexibility in packaging and shipping.

In general, steer-by-wire system is expected to provide a better operation platform of lateral motion controllers.

#### 4. ADVANCES IN VEHICLE LATERAL MOTION MONITORING

##### 4.1 Vehicle Lateral Motion Related Sensory

Since a vehicle is a highly complex system that contains varied mechanical, electronic and electromechanical elements, numerous sensors were designed and employed to measure vehicle movement information. Concentrated on vehicle lateral motion control, position and movement information of a vehicle need to be precisely measured.

In [48], sensors for lateral control are classified into two types: infrastructure-based and infrastructure-independent. Examples of the infrastructure-based systems include discrete magnetic reference markers and continuous magnetic tape [48]-[50]. The infrastructure-independent methods use, for example, global positioning system (GPS) [51]-[53], inertia position system (INS) [54]-[59], or vision system for sensing [60]-[64]. But, these infrastructure-independent systems still rely on infrastructure in the sense of reliable roadway markings in the former case, and a reliable and accurate roadway geographical information system (GIS) database.

Global positioning system (GPS), inertial navigation system (INS) and their combinations attract great interest in the last decade. The position and velocity of a vehicle can be directly measured by using global position systems. It had been proven in several literals including [51]-[57] that sideslip angle, yaw rate, heading angle, position, and side displacements can be indirectly estimated with cooperated inertial navigation system and global position systems.

Moreover, the newly developed fiber optic gyroscopes (FOP) are capable to measure sideslip angle, yaw rate, heading angle straightforward with amazing high accuracy [58]-[59]. But current FOPs usually require considerable installing and maintaining cost, which prevents their widely application in automotive industry.

Magnetic sensing is another promising technology that has been developed recently for the purposes of vehicle position measurement and guidance. By using either magnetic tape or magnetic markers, vehicle position displacement can be gotten as well some other useful information [48]-[50].

Vision sensors can also be employed to measure vehicle displacement. The standard deviation of the error in the position detection can be made less than 0.1 m. However, some special lane detection algorithms need to be used, which is time-consuming for the onboard computers. A more convenient approach is to use laser sensor. For example, the offset between the vehicle and road curb can be directly obtained by using the laser sensor proposed in [18]. However, measure performance of these two methods is more vulnerable to environment disturbances than that of the above two techniques, i.e. the performance of CCD sensor is sensitive to fog.

Besides, other vehicle lateral motion related parameters and variables can also be directly measured or indirectly estimated [65]-[66]. For instance, how to estimate vehicle inertia tensor is discussed in [66]. In the rest of this paper, we assume that all the information needed has already been accurately measured.

## 4.2 Vehicle Lateral Motion Observer Design

In many recent approaches, not all the vehicle characteristics are instantly measured due to high cost or some other reasons. Instead, several special observers are used to reconstruct the needed information. In literals, these observers were also called virtual sensors.

For example, knowledge of sideslip angle, yaw rate and lateral velocity is essential in vehicle control, but is difficult to obtain directly. In 1997 and 1999, Kiencke etc. proposed a linear observer and a nonlinear observer using reduced order bicycle model in [67] and [68]. Soon after that, Venhovens and Naab used a Kalman filter in [69] for a linear vehicle model in 1999. Huh etc. constructed the monitoring system in [70] based on KFMEC (Scaled Kalman Filter with Model Error Compensator) technique to improve the robustness of ordinary Kalman filters. In [71], Zhang, Xu and Rachid showed the feasibility of a special sliding mode observer for vehicle lateral motion. Similar conclusions were reached by Perruquetti and Barbot in [72].

To filter out the unexpected effect of disturbances from the observer output, different robust design methods had been introduced in Luenberger observer construction. In [73]-[75],  $H_\infty$  filter theory was employed to design the optimal observers to resist disturbance for reduced order bicycle model. In [76],  $H_\infty$  loop shaping was used for observer design of the linearized lateral motion model of a single-unit HDV (tractor-semitrailer type vehicle). In [77], the equivalent  $H_\infty$  LMI design method was applied in

Luenberger observer and fault detection filter design.

The above "bicycle model" (7) and (8) can be written into canonical form as

$$\dot{x} = Ax + Bu + Ew \quad (16)$$

where  $x = [\beta \quad r \quad \psi \quad y_f \quad y_r]^T$  is the state variable,  $u = [\delta_f \quad \delta_r]^T$  is the control input, and  $w = [\rho_{ref} \quad f_w]^T$  is the disturbance.  $A$ ,  $B$  and  $E$  are the corresponding system matrices.

The measurement output  $y$  can be formulated as

$$y = Cx + Du \quad (17)$$

Usually, the linear Luenberger observer is formulated as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu - L'(y - \hat{y}) \\ \hat{y} = C\hat{x} + Du \end{cases} \quad (18)$$

where  $\hat{x}$  denotes observer state,  $\hat{y}$  denotes observer output.  $L$  is the observer matrix.

Its performance index can be specified as

Minimize the  $H_\infty$  norm of the transfer function matrix from  $w$  to  $e$ . Or equivalently Choose the smallest  $\gamma > 0$  such that

$$\int_0^{+\infty} e^T e dt \leq \gamma^2 \int_0^{+\infty} w^T w dt \quad (19)$$

This leads to the following LMI design problem

Min  $\gamma$  with  $\gamma > 0$ ,  $\mu > 0$  and moderate  $\tau > 0$  such that

$$\left[ \begin{array}{c|c} PA + A^T P + X' C + C^T X'^T + I & PE \\ \hline E^T P & -\gamma^2 \end{array} \right] < 0 \quad (20)$$

with the observer matrix

$$L' = P^{-1} X' \quad (21)$$

Most above observers utilized the accurate dynamic model using nominal values including tire concerning stiffness, vehicle mass and moment of inertia and distances between center of mass and tires. Thus, these observers depend on an accurate knowledge of these parameters, and are affected by variations in them. For instance, Stephant, Charara and Meizel pointed out in [15] that the speed of center of gravity is not an indispensable variable.

One method to solve this problem is to choose the estimation method without utilizing the vehicle dynamic model, i.e. the observers proposed in [79] and [80]. However, the non model-based observers are hard to apply along with steering control system.

Another method is to introducing robust observer that is not sensitive to system parameter changes or adaptive observer that can change itself according to parameter

change. In [81], a robust observer was developed by including an extra term and adopting the Lyapunov stability theorem.

The system dynamics is modified as below considering model uncertainty and nonlinear properties

$$\dot{x} = (A + \Delta A)x + Bu + Ew \quad (22)$$

where  $\Delta A$  denotes the variance matrix that is determined by variance of mass, velocity, tire-road friction coefficients and nonlinear characteristics. And the norm of  $\Delta A$  is assumed to be bounded as

$$\|\Delta A\|_2 \leq \varepsilon \quad (23)$$

In [81], the proposed robust observer for system (22) as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu - L(y - \hat{y}) + \tau\alpha \\ \hat{y} = C\hat{x} + Du \\ \alpha = \frac{\varepsilon^2 \hat{x}^T \hat{x}}{(y - \hat{y})^T (y - \hat{y})} P^{-1} C^T (y - \hat{y}) \end{cases} \quad (24)$$

where  $\tau$  is a positive real scalar that needs to be determined.

This leads to an Algebra Riccati Equation (ARE) problem

Min  $\gamma$

with  $\gamma > 0$ ,  $\lambda > 0$  and moderate  $\tau > 0$  such that

$$\begin{bmatrix} PA + A^T P + XC + C^T X^T + & | & PE \\ (1 + \lambda\varepsilon^2)I + (1/\lambda + 1/\tau)2P^2 & & \\ \hline E^T P & | & -\gamma^2 \end{bmatrix} < 0 \quad (25)$$

with the observer matrix

$$L = P^{-1} X \quad (26)$$

It is proven in [81] that this nonlinear observer maintains the good disturbance rejection property that derived form [78]-[75], while provides tolerance to model variance as the observer too.

In [15], Stephant etc. compared four different observers including linear Luenberger observer and three nonlinear observers: extended Luenberger observer, extended Kalman filter and sliding-mode observer. Based on simulation results and practical experiments, they showed that all four observers can yield acceptable estimation results if the observer's parameters are appropriately assigned. And the later two can yield relatively better performance.

## 5. ADVANCES IN VEHICLE LATERAL MOTION CONTROL

Early as 1969, Kasselmann and Keranen [7] developed an active steering system based on feedback from a yaw

rate sensor. With continuous efforts, people gradually realized that the difficulties of steering control mainly lie in the following five aspects:

- 1) how to avoid skidding during steering, which is one frequently encountered hazardous situation for green hand drivers;
- 2) how to reject the disturbance caused by wind or some other reasons;
- 3) how to deal with vehicle dynamics uncertainty and variation;
- 4) how to handle actuator rate limits during steering;
- 5) how to handle time delay exists in feedback block during steering.

To answer these five questions, numerous designed methods had been proposed in the last three decades. Scanning the previous reported efforts, we find that varied linear robust controllers attract continuous interest through the last two decades. These studies provide us some basic solutions to deal with skidding, disturbance, parameter uncertainties and steer angle saturation. Based on these results, different sliding model controller, fuzzy controller and adaptive controller received increasing efforts as time goes on. These nonlinear controllers usually outperform linear ones in several different aspects. In this paper, we will mainly examine these four controllers.

### 5.1 Frequency Domain Robust Steering Controllers

Originated in the later 80s, frequency domain robust design techniques soon became and remained as one of the most important techniques in field of vehicle lateral motion control. It had been proven to be a practical and efficient approach by lots of literature [82]-[103].

One direct idea to avoid skidding is to remove the influence of  $r$  on the lateral acceleration. The lateral and yaw motions of a car with active steering is decoupled by Ackermann in [82]. It was proved that for an ideal longitudinal mass distribution, the decoupling by yaw rate feedback is robust with respect to uncertain nonlinear tire side force characteristic, velocity and vehicle mass. But Ackermann later showed that this was not a simple and cheap control system since it requires measuring longitudinal velocity of vehicle  $v_x$  ( $v_x \approx v \sin \beta$ ), yaw rate  $r$  and its derivative, slip angle  $\beta$  simultaneously. Thus, a practical controller was proposed in [83], in which only  $v_x$  needs to be measured. This simplified controller was proved to have similar steady-state behavior to a car.

It was further shown that additional feedback of the yaw rate  $r$  leads to a significant reduction of the deviation from the guideline in nearly all driving maneuvers compared to earlier controllers which used solely feedback of the deviation  $y_f$ . Therefore, a feedback controller with

respect to both  $r$  and  $y_f$  was proposed as

$$\dot{\delta}_f = u_f - k_r \cdot r \quad (27)$$

where  $u_f$  was a determined as

$$\frac{u_f(s)}{y_f(s)} = \omega_c^2 \frac{k_{DD}s^2 + k_Ds + k_P}{s^2 + 2D\omega_c s + \omega_c^2} \quad (28)$$

and inside the bandwidth  $\omega_c$ ,  $k_P$  denotes a proportional part,  $k_D$  denotes a differential part and  $k_{DD}$  denotes the double differential part.

As revealed in [82] and [83], using careful poles and zeros assignment, the system can be well stabilized. Besides it, the redundancy of the design parameters can be used to count off the variance of vehicle dynamics. For instance, the  $\Gamma$ -stability boundaries was analyzed for parameters  $(k_D, k_{DD})$  in [83].

Moreover, a generic control law for robust decoupling of lateral and yaw motion by yaw-rate feedback to front-wheel steering was derived in [85]. It showed that ideal steering dynamics were able to be achieved by velocity scheduled lateral acceleration feedback to front-wheel steering. For robust yaw stabilization a velocity-scheduled yaw-rate feedback to rear-wheel steering is given, by which the linearized system gets velocity-independent yaw eigenvalues. In [86], it was further proven that decoupling by yaw rate feedback is robust with respect to uncertain nonlinear tire side force characteristic, velocity and vehicle mass, if an ideal longitudinal mass distribution is assumed.

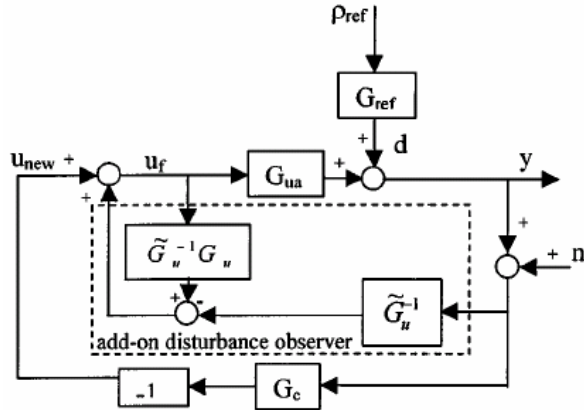


Fig.6. Diagram of system architecture with add-on disturbance observer [89].

In the last 90's,  $H_\infty$  robust analysis method was introduced into steering controller design field to reduce the unexpected effect of wind disturbance.  $H_\infty$  theory is constructed to handle the deterministic disturbance model consisting of bounded energy (square-integrable)  $L_2$  signals and allows controller design for narrow-band disturbance rejection, Francis [87] and Zames [88]. In [89], Guvenc, Bunte and Odenthal etc. designed a disturbance observer in Fig.6, whose model regulation capability

allows the specification and achievement of desired yaw dynamics. In the proposed integrated control model,  $G_{ref}$  was the transfer function from disturbance torque  $\rho_{ref}$  to yaw rate  $r$ .  $\tilde{G}_u$  is the nominal system model and  $G_u$  is the un-modeled dynamics.

It was pointed out in [89] that the model regulation and disturbance rejection property of this proposed observer can be considered as a special  $H_\infty$  loop shaping for path following. The filtering effect is chosen to satisfy the frequently used loop shaping constraint

$$|W_s S| + |W_T T| < 1, \text{ for } \forall \omega \quad (29)$$

where  $W_s$  and  $W_T$  are the sensitivity function weight and the complementary sensitivity function weight respectively.  $S$  and  $T$  denote the sensitivity function and transfer function, respectively.

Equivalently, Mammari etc. developed several two degree of freedom (2DOF) steering controller in [90]-[92] using the  $H_\infty$  loop shaping technique, see Fig.7. The design task is directly assigned as finding a robust feedback control  $\tilde{K}$  to guarantee the stabilizing of system and minimize energy bound of the transfer function from  $\rho_{ref}$  to the pre-selected measurement output  $z_1$ ,  $z_2$  and  $z_3$ . Based on the works of Kuzuya and Shin that was reported in [93], these robust 2DOF steering controller can be easily digital implemented.

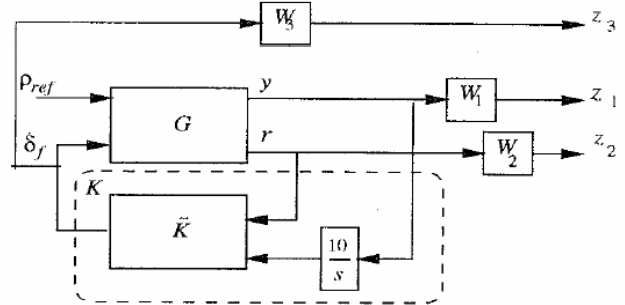


Fig.7. Diagram of two degree of freedom (2DOF) steering controller [90].

Simulations and experiments pointed out that the steering angle rate actuator saturation forms a major limitation of performance. In some literals, the saturation properties of the steering actuator were further examined. For example in [94], the actual steering control considering steering actuator rate limits is shown in Fig.8.

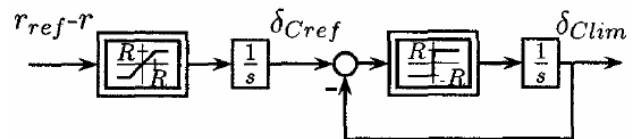


Fig.8. Steering controller considering actuator saturation [94].

By analyzing the uncertainty of the parameters  $v$  and  $\mu$ , one typical operating range  $Q$  for  $v$  and  $\mu$  is shown in Fig.9. The modified transfer function can be formulated as

$$G_{\delta_f-r}(s) = \frac{r(s)}{\delta_f(s)} = \frac{\mu v (f_1 v s + f_0 \mu)}{e_2 v^2 s^2 + e_1 \mu v s + (e_{01} \mu^2 + e_{02} \mu v^2)} \quad (30)$$

where

$$\begin{aligned} f_0 &= c_f c_r (l_f + l_r), \quad f_1 = c_f m l_f \\ e_{01} &= f_0 (l_f + l_r), \quad e_{02} = (c_r l_r - c_f l_f) m \\ e_1 &= m (c_r l_r + c_f l_f) (l_f + l_r), \quad e_2 = m^2 l_r l_f \end{aligned}$$

It was revealed that the undesirable limit cycles caused by saturations were analyzed by a describing function approach in combination with the representation of limit-cycle-free regions in the parameter plane of velocity  $v$  and road/tire friction coefficient  $\mu$ , see Fig.9. The results were formulated in terms of required actuator bandwidth that achieves robustness in the entire operating range. It turned out that the use of a fading integrator can reduce the required actuator bandwidth. Based on similar ideas, a compensator with high order was investigated in [90] to achieve better performance.

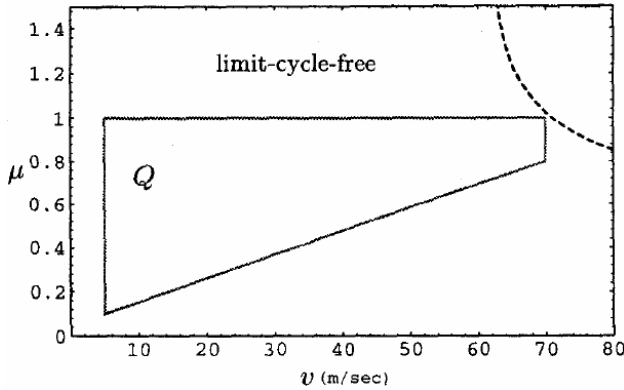


Fig.9. The designed operating range  $Q$  for  $\mu$  [94].

There were some other approaches based on  $H_\infty$  theory reported in [95]-[103]. Most of them were mainly devoted to find a good balance between steering performance and wind disturbance rejection. Specially in [99]-[100], the effect of time delay in steering feedback controller is studied. Constrained by the length, detailed discussions are neglected here.

## 5.2 Time Domain Robust Steering Controllers

Recently, time domain robust design technique was used in vehicle lateral control too [104]-[107]. Although time domain robust design is intrinsically equivalent to frequency domain robust design, the obtained controllers differ in many aspects.

In [107], the steering system (7) was modified as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{\psi} \\ \dot{y}_f \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & b_{11} \\ a_{21} & a_{22} & 0 & 0 & b_{21} \\ 0 & 1 & 0 & 0 & 0 \\ v & l_{fs} & v & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ \psi \\ y_f \\ \delta_f \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_f + \begin{bmatrix} 0 \\ 0 \\ -v \\ -v l_{fs} \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{f}_w \end{bmatrix} \quad (31)$$

which can be further reformulated as

$$\begin{cases} \dot{x} = Ax + Bu' + Ew \\ z_\infty = C_\infty x \\ z_1 = C_1 x \\ z_u = u' = \text{sat}(Kx) \end{cases} \quad (32)$$

where  $z_\infty \in C^{n_\infty}$  is the introduced  $H_\infty$ -performance output,  $z_1 \in C^{n_1}$  is the introduced  $L_1$ -performance output,  $z_u \in C^{n_u}$  is the auxiliary performance output for bounded control input.  $C_\infty$  and  $C_1$  are ride performance measurement matrices.

The proposed mixed  $L_1 / H_\infty$  observer-based state feedback controller was designed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + LC(y - C\hat{x}) \\ u_i = -\frac{\lambda}{2} B_i^T Q^{-1} \hat{x} \end{cases} \quad (33)$$

Notice that in (2)-(3), the safe ride requirement on  $y_f$  and  $\delta_f$  can be measured and guaranteed by choosing

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } C_\infty = I \quad (34)$$

The actuator saturation limit on  $\delta_f$  is hold by keeping

$$u' = \text{sat}(Kx) = \begin{cases} Kx, & |Kx| \leq \dot{\delta}_{\max} \\ \text{sign}(Kx) * \dot{\delta}_{\max}, & |Kx| > \dot{\delta}_{\max} \end{cases} \quad (35)$$

It was proven that the safety requirement will be met as

$$\begin{aligned} \|u_1\|_\infty &\leq \gamma_{u,1} w_{\max} \leq \dot{\delta}_{f \max} \\ \|z_{1,1}\|_\infty &\leq \sqrt{2} \gamma_{1,1} w_{\max} \leq \delta_{f \max} \\ \|z_{1,2}\|_\infty &\leq \sqrt{2} \gamma_{1,2} w_{\max} \leq y_{f \max} \end{aligned}$$

if the following LMI can be satisfied

$$\begin{bmatrix} AQ + QA^T - \lambda BB^T & E & QC_\infty^T \\ E & -\gamma_\infty^2 & 0 \\ C_\infty Q & 0 & -I \end{bmatrix} < 0 \quad (36)$$

$$\begin{bmatrix} AQ + QA^T + \alpha Q - \lambda BB^T & E \\ E^T & -\alpha \end{bmatrix} \leq 0 \quad (37)$$

$$\begin{bmatrix} 4Q & \lambda B_i \\ \lambda B_i^T & \gamma_{u,i}^2 \end{bmatrix} > 0 \quad (38)$$

and

$$\begin{bmatrix} Q & QC_{1,j}^T \\ C_{1,j}Q & \gamma_{1,j}^2 \end{bmatrix} > 0 \quad (39)$$

Based on the above results, [108] studied the relationship between navigation speed and the corresponding cruise offset with given steering curvature. As a result, an estimation method for maximum safe speed regarding a curve or a lane change trajectory is provided, too.

### 5.3 Sliding Mode Steering Controllers

Sliding mode steering controller is another frequently used steering controller [22], [82]-[114]. Generally speaking, the basic idea of sliding mode control is to restrict the state space trajectories of the dynamic system to a manifold called “sliding manifold” which is usually denoted by  $S = 0$ . This is achieved by directing the system trajectories towards this manifold “from both sides”.

In [82], the sliding manifold was chosen as

$$S = c\Delta r + \Delta \dot{r} \quad (40)$$

where  $c > 0$  is a constant gain that determines system behavior once the motion of system (7) or (8) has been restricted to the neighborhood of the manifold  $S = 0$ .

The structure of the proposed controller was shown in Fig.10, in which the system ideal feedback control strategy is written as

$$r_d = -\frac{1}{l_s} [v(\beta + \psi) + Ky] \quad (41)$$

where  $K > 0$  determines the desired rate of decay of  $y$ .

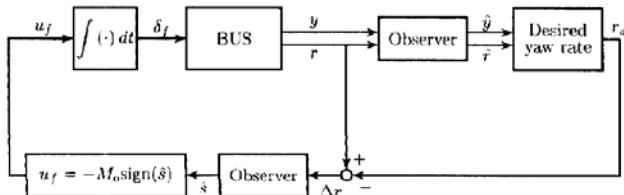


Fig.10. Diagram of a typical sliding mode steering controller [82].

Notice both the states  $\beta$  and  $\psi$  are unknown, the following observer (42) and (43) was introduced for estimating these two state variables

$$\dot{\hat{y}} = \hat{q} + l_1 r + l_1 \bar{y}, \quad l_1 > 0 \quad (42)$$

$$\dot{\hat{q}} = l_2 \bar{y}, \quad l_2 > 0 \quad (43)$$

Thus the actual feedback control was obtained as

$$r_d = -\frac{1}{l_s} [\hat{q} + K\hat{y}] \quad (44)$$

and the feedback control was chosen as

$$u_f = -M_u \text{sign}(S) \quad (45)$$

where  $M_u$  is the available steering angle rate.

To further improve passenger comfort, it was proved to be advantageous to replace linear term  $K\hat{y}$  in (44) by a

saturation function

$$r_d = -\frac{1}{l_s} \left[ \hat{q} + \lambda \frac{\hat{y}}{\sqrt{\hat{y}^2 + \varepsilon}} \right], \quad \lambda > 0, \quad \varepsilon > 0 \quad (46)$$

and the feedback control can be substituted by a continuous approximation as

$$u_f = -M_u \frac{S}{\sqrt{S^2 + 0.0001}} \quad (47)$$

Comparing to the above linear controllers, it was proven in [82] that this sliding mode controller yields smaller deviations from the guideline and it has a more oscillatory behavior that shows up particularly in the lateral acceleration and in the steering angle rate, but not in the derivations from the guideline. Regarding settling times, there are no significant differences between the two controllers.

There were several other sliding mode steering controllers which chose different sliding manifold [109]-[113]. In [109] the lateral and longitudinal control of vehicle was studied using a PID typed sliding surfaces, whose stability was proven using Lyapunov theory. In [110], a velocity related sliding mode controller was proposed to deal the input couple problem. Another special sliding mode integral action controller and the corresponding sliding mode observer are used to enhance vehicle stability in a split- $\mu$  maneuver in [22]. In [111], it was shown that the sliding mode controller can also be used to deal with the nonlinear front steering model considering track force control. Moreover, sliding mode steering controller is an important approach in tractor-trailer vehicles lateral motion control [112].

### 5.4 Adaptive Steering Controllers

Because of its capability of handling model uncertainty and parameter variation, adaptive steering controllers achieve continuous interest in the last twenty years. Generally, there are two different methods: model concentrated approaches [115]- [120] and non-model concentrated approaches [121]-[123].

In [115], Brennan and Alleyne proposed a steering controller based on model reference control (MRC) with a modification based on rejection of known disturbance dynamics. Model reference control was initiated by Astrom and Wittenmark in 1997 [116]. This method was shown in [115] to be effective for steering control systems consist of dynamic uncertainty and disturbance. In [117], a special adaptive observer was proposed to deal with system parameters variations. In [118], an adaptive rule was proposed to make the controller flexible with velocity change. However, how to keep the balance between steering control precision and cost of implementation facility still needs to further discussion.

In [121]-[123], the concepts of Selected Adaptive Critic

(AC) and Dual Heuristic Programming (DHP) were used to design steering controller. Selected adaptive critic methods are known to be capable of designing (approximately) optimal control policies for non-linear plants (in the sense of approximating Bellman Dynamic Programming). The present research focuses on an AC method known as dual heuristic programming. There were lots of issues related to the pragmatics of successfully applying the AC methods.

In [121], a straight forward utility function to meet these requirements would take the following form:

$$\delta_f = -Ay_{f_{error}}^2 - B\dot{y}_{f_{error}}^2 - Cv_{y_{error}}^2 - D\dot{v}_{y_{error}}^2 \quad (24)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  were determined from programming to indicate the human designer's judgment about the relative importance of each term, according to desired plant response characteristics (e.g., the derivative terms encourage more "damped" responses). In [123], it was further shown that DHP can be employed to optimize fuzzy steering controllers.

### 5.5 Fuzzy Steering Controllers

Fuzzy set theory and Fuzzy inference was first presented by Zadeh in 1965. Recently, some new approaches take advantage of Fuzzy inference to avoid addressing complex vehicle dynamics. Moreover, these approaches were proven to be able to incorporate and utilize human steering skills to improve the automatic driving performance [124]-[135].

For example, a direct Fuzzy control strategy was proposed by Brown and Hung 1994 in [125] for a 4WS car model. The corresponding Fuzzy control rule is something as

```
"IF
  Yaw_Rate_Error is Negative_Large AND
  Front_Slip is Negative_Large
THEN
  Command_Front_Steering_Angle is Positive_Large"
```

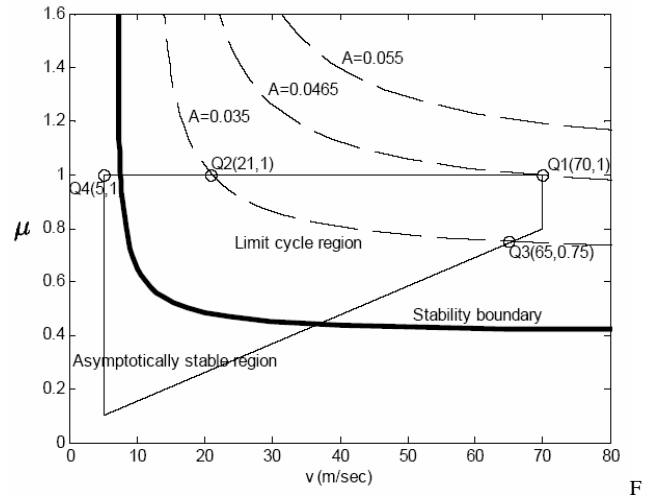
The front steering command and the rear steering command were designed to cooperate appropriately to avoid skipping in [125]. Assigning appropriate fuzzy membership function and rule table, Brown and Hung claimed that the 4WS car using this Fuzzy controller was quite robust to wind gusts and road perturbations.

There were several other directly fuzzy controllers reported in [126]-[131]. However, it should be remarked that most of them were still heuristic controllers, for which no theoretical stability proof had been presented.

In some recent literatures, the proposed fuzzy controllers were constructed as follows. First, an optimal steering controller was constructed for each local model, which indeed constructed a part of the fuzzy model. Then, local controllers were combined using fuzzy rules to form a fuzzy logic controller. Therefore, the performance of the

fuzzy controller can be analyzed using linear matrix inequalities (LMI) or algebra Riccati equation (ARE).

In [132]-[134], these methods were shown to be effective with both theoretical analysis and simulations. In [135], the fuzzy describing function analysis and traditional frequency domain method are applied to determine the stability condition when steering system has perturbation or adjustable parameters. And the stable boundary of the fuzzy controller can be finally determined with the obtained operating range  $Q$  similar to what discussed in [94]; see Fig.9 and Fig.11.



ig.11. The designed operating range  $Q$  for  $\mu$  [135].

There were some special nonlinear steering controllers proposed recently, i.e. [136]-[137]. Usually their stability was guaranteed by Lyapunov theory. However, most such approaches did not carefully consider model uncertainty robustness and actuator saturation cases.

In [138], a simple proportional controller was compared with  $H_\infty$  robust controllers, fuzzy controller and adaptive controller. This proportional feedback was found to yield the largest offset with respect to other controller, although it was only slightly affected by the wind force. On the other hand, the self-tuning regulator presents the smallest errors. The responses of  $H_\infty$  and fuzzy controllers are comparable in most tests regarding to self-tuning regulator. Although these conclusions were made for special controllers, they were widely expected to be true in general situations.

## 6. CONCLUSIONS

The recent trends of research on development of vehicle steering control are reviewed in this paper. The focus was on vehicle controller design techniques. Research advances in vehicle dynamic modeling, steer-by-wire, onboard sensory, and observer design were also briefly investigated.

However, due to length limits and their premature, some

important contents leave untouched in this paper. For instance,

- 1) the relationship between vehicle longitudinal and lateral motion should be further analyzed, although it was proven that these two motions can be roughly decoupled [139]-[142]. Especially how to guarantee ride safety when both steering and braking occurs is an important problem that should be carefully studied [5], [142];
- 2) the effect of sensor installment and measurement error needs further discussions [143];
- 3) Although great efforts had been put in steer-by-wire research, how to make intelligent steering controller cooperate appropriately with driver command still remains as a difficult problem to be fully conquered. Some previous works [144]-[146] revealed that this problem is quite complex, in which both drivers' characteristics, feelings, and driving status should be further analyzed;
- 4) rollover avoidance using active steering control is gaining more attentions recently. Some promising results had been reported in [147]-[152]. A detailed discussion will be carried out in our coming paper.
- 5) moreover, fault tolerant steering control and related fault detection algorithms are archiving increasing considerations recently [153]-[159]. With rapidly increasing demands on driving safety, a boom in this research field is expected to appear in the near future.

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