

# An Approach to Linkage Stability Analysis for in Mobile Ad hoc Network by Markov Jump Theory<sup>1</sup>

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**Abstract**— Since mobile nodes in a mobile ad hoc network (MANET) may spread in an arbitrary manner, one of the fundamental issues in such a network is the linkage problem among mobile nodes. In this paper, we analyze the linkage stability for special cases in mobile ad hoc network based on Markov jump theory. We find that if some conditions are satisfied, the solutions for the proposed linkage models are almost surely exponentially stable. Some examples are described to illuminate our theoretic results.

**Index Terms**—Ad Hoc Network, Stability, Markov Jump Theory

## 1. INTRODUCTION

Ad hoc networks are characterized by dynamic topology due to node mobility, limited bandwidth and limited battery power of nodes. A Stochastic Petri net-based approach to modeling and analysis of ad hoc network is presented in the work [1]. The problem of wireless vulnerabilities and limited physical security in the design of such network is considered in the work [2]. This work also explores the new approaches trying to solve current problems and point out the future research directions. Spread spectrum (SS) is considered as an access technology for MANETs. The work [3] proposes an SS Medium access control protocol with dynamic rate and collision avoidance called Dynamic-Rate and Collision Avoidance for MANETs. Intervehicle communication is a key technique of intelligent transport systems. A new clustering technique for ad hoc vehicle networks is proposed in the work [4]. A multi-hop wireless ad hoc network is a collection of nodes (devices) that communicate with each other without any established infrastructure, centralized control and pre-determined organization of available links [5,6]. In a MANET, one of the fundamental issues is the node linkage problem. Mobility plays a crucial role in it. Relative node movements can create or break links, and change the network topology. The work [7] and [8] models the random evolution networks by mean-field theory, and [9] describes the node local linkage processes of the wireless sensor networks by continuum theory. The objective of our modeling approach is to create an enhanced model that describes the local processes of the evolution of the network, incorporating the addition of new links, rewiring of existing links, deleting of existing links, etc. In mobile wireless sensor networks or

MANETs, the links among nodes are subject to node movement, energy decrease of nodes, network density changes, power coverage changes, etc. Systems with those characteristics may be modeled as hybrid states. Markov jump theory [10] is an important modeling tool for hybrid ones. For a finite state Markov jump system, the finite Markov chain governs the transition from one logical state to another. The work [11] creates a linkage model based on Markov jump theory for MANET and generalizes the model established in [9]. In this paper, we discuss the stability of solutions in three special cases of the model established in our prior work [11] and give some examples.

The remainder of the present paper is arranged as follows. The basic model is formulated in Section 2 for ad hoc network and some notations are presented. In Section 3, linkage models for special cases are given, and linkage analysis including almost surely exponential stability of solutions is performed. Some numerical examples in Section 4 are provided to verify the results. Finally, some concluding remarks are given in Section 5.

## 2. BASIC MODEL

In our concerned MANET, we assume that the number  $N$  ( $N \geq 2$ ) of nodes is constant. In other words, there are neither new nodes joining the system nor existing nodes leaving the system. According to the model for MANET [11], we have a general model in the form of a Markov jump differential equation for node  $k$  with  $x_k$  links, where node  $k$  is randomly selected and  $x_k \in \{1, 2, \dots, N-1\}$ .

$$\frac{dx_k(t)}{dt} = f(x_k(t), t, r(t)) \quad (1)$$

where  $f(x_k(t), t, r(t))$  is the linkage variant rate of node  $k$  and  $r(t) \in S = \{1, 2, 3, 4\}$ . Then states 1, 2, 3 and 4 forms a Markov chain with generator  $\Gamma = (\gamma_{ij})_{4 \times 4}$  given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j \end{cases} \quad (2)$$

where  $\Delta > 0$ . Here  $\gamma_{ij} \geq 0$  is the transition rate from state  $i$  to state  $j$  if  $i \neq j$ , while  $\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij}$ .

In the following, for a node  $k$  that is arbitrarily selected in a MANET, we introduce four states.

State 1. The state of deleting  $m_1$  links with probability  $r$  ( $0 \leq r < 1$ )

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$$f_1(x_k(t), t, 1) = -rm_1 Q_1(x_k(t)) - rm_1 A_k \quad (3)$$

The first term corresponds to the decreasing linkage because other nodes connected with node  $k$  select to remove their links with it, while the second term corresponds to the case that the node itself selects to remove one of its links, where  $A_k = \sum_{\text{all } x_k \text{ link } l_{kj}} Q_1(x_j(t)) / x_j(t)$ ,

where  $x_j$  denotes node  $j$  currently associated with links, and  $Q_1(x_k(t))$  denotes the probability that a node  $k$  associated with  $x_k(t)$  links at time  $t$  is selected.

State 2. The state of rewiring  $m_2$  links with probability  $q$  ( $0 \leq q < 1$ )

$$f_2(x_k(t), t, 2) = -\frac{qm_2}{N} + qm_2 Q_2(x_k(t)) \quad (4)$$

The first term corresponds to the decrease of the number of links with the node from which the links is removed, and the second term corresponds to the increasing connectivity of the node that the link is reconnected to and based on probability  $Q_2(x_k(t))$ . Note that  $Q_2(x_k(t))$  is defined similarly to  $Q_1(x_k(t))$ .

State 3. The state of adding  $m_3$  links with probability  $p$  ( $0 \leq p < 1$ )

$$f_3(x_k(t), t, 3) = \frac{pm_3}{N} + pm_3 Q_3(x_k(t)) \quad (5)$$

The first term is due to the random selection of the starting point of a link, while the second term corresponds to the end point selection based on probability  $Q_3(x_k(t))$ . Note that  $Q_3(x_k(t))$  is defined similarly to  $Q_1(x_k(t))$ .

State 4. The state of unchangeable links with probability  $w$  ( $0 \leq w < 1$ ) where  $p + q + r + w = 1$ .

$$f_4(x_k(t), t, 4) = 0 \quad (6)$$

**Remark 1** Apparently, for any node in a MANET, it may be called the steady-state because the number of its connections is not changed when the system in states 2 and 4. On the contrary, states 1 and 3, due to the decrease or increase in the number of links, can be called non-steady-state. Accordingly, for the integrated system made up of four states, it is the focus of this paper that we will analyze the stability by Markov jump theory.

### 3. STABILITY ANALYSIS FOR SPECIFIC MODELS

In the initial model,  $Q_1(x_k(t))$ ,  $Q_2(x_k(t))$  and  $Q_3(x_k(t))$  denote the probability that a link or a node is selected in states 1-3, respectively. In the following we discuss the representation of these probabilities for three different cases.

#### 3.1 Stability of Solutions for Specific Linkage Models

##### 3.1 Specific Linkage Model One

In the first model, new links preferentially point to

popular nodes, while the more links the node is associated with, the higher the probability that the nodes themselves remove a link.

According to this case and [9],

$$Q_2(x_k) = Q_3(x_k) = \frac{x_k + 1}{\sum_{i=1}^N (x_i + 1)} \quad (7)$$

$$Q_1(x_k) = \frac{x_k}{\sum_{i=1}^N x_i} \quad (8)$$

We have

$$A_k = \sum_{\text{all } x_k \text{ links } l_{kj}} \frac{Q_1(x_j)}{x_j} = \sum_{\text{all } x_k \text{ links } l_{kj}} \frac{x_j}{\sum_{i=1}^N x_d} \frac{1}{x_j} = Q_1(x_k),$$

where  $x_k$ ,  $x_i$ ,  $x_j$  and  $x_d$  denote nodes  $k$ ,  $i$ ,  $j$  and  $d$  currently associated with links, respectively. "all  $x_k$  links  $l_{kj}$ " denote all links with  $k$ .

Consider only states  $r(t) = 1$  and  $r(t) = 3$  contributing to the total number of links in the whole network at time  $t$ .

From [9],  $\sum_{i=1}^N x_i$  denotes the total number of links in the whole network at time  $t$ . Only states 1 and 3 contribute to  $\sum_{i=1}^N x_i$ , and therefore we have  $\sum_{i=1}^N x_i = 2(pm_3 - rm_1)t$ .

Apparently, the maximum of  $\sum_{i=1}^N x_i$  is

$$C_N^1 \cdot C_{N-1}^1 = N^2 - N \text{ in a network with } N \text{ nodes.}$$

**Lemma 1**  $\forall (x_k, t, i) \in R^+ \times R^+ \times S$ ,

$$x_k f(x_k, t, i) \leq \alpha_i |x_k|^2 \quad (9)$$

where

$$\alpha_1 = -\frac{2}{N^2 - N} rm_1, \alpha_2 = \frac{1}{N} qm_2, \alpha_3 = \frac{3}{N} pm_3, \alpha_4 > 0, \text{ and}$$

here we set  $\alpha_4 = w$  for the convenience. ■

**Proof** According to the maximum of  $\sum_{i=1}^N x_i$ , the

maximum of  $(pm_3 - rm_1)t$  is  $\frac{N^2 - N}{2}$  in a network with  $N$  nodes. We have

$$x_k f(x_k, t, 1) = x_k \left[ -\frac{2rm_1 x_k}{\sum_{i=1}^N x_i} \right] \leq -\frac{2rm_1}{N^2 - N} x_k^2$$

$$\begin{aligned}
 x_k f(x_k, t, 2) &= x_k \left[ -\frac{qm_2}{N} + qm_2 \frac{x_k + 1}{\sum_{i=1}^N x_i + N} \right] \\
 &\leq \frac{qm_2}{N} x_k^2 \\
 x_k f(x_k, t, 3) &= x_k \left[ \frac{pm_3}{N} + pm_3 \frac{x_k + 1}{\sum_{i=1}^N x_i + N} \right] \\
 &\leq \frac{3pm_3}{N} x_k^2 \\
 x_k f(x_k, t, 4) &= 0 \leq wx_k^2
 \end{aligned}$$

Therefore, we have

$$x_k f(x_k, t, i) \leq \alpha_i |x_k|^2. \quad \blacksquare$$

**Remark 2** This lemma finds the appropriate values for  $\alpha_i$  in order to meet the condition (9). (9) is valid for any node at any time and state.

**Lemma 2**  $\forall (x_k, t, i) \in R^+ \times R^+ \times S$ ,

$$|f(x_k, t, i)| \leq K |x_k|, \quad t \in [t_0, \infty) \quad (10)$$

where  $K = \max \left\{ \frac{rm_1}{(pm_3 - rm_1)t_0}, \frac{qm_2}{2}, pm_3(N+2), 1 \right\} \quad \blacksquare$

**Proof** If we consider that there are no links at the beginning, then the following condition should be satisfied,  $pm_3 - rm_1 > 0$ . So

$$\begin{aligned}
 |f(x_k, t, 1)| &= \left| \frac{2rm_1 x_k}{\sum_{i=1}^N x_i} \right| \leq \frac{rm_1}{(pm_3 - rm_1)t_0} |x_k| \\
 |f(x_k, t, 2)| &= \left| -\frac{qm_2}{N} + qm_2 \frac{x_k + 1}{\sum_{i=1}^N (x_i + 1)} \right| \leq \frac{qm_2}{2} |x_k| \\
 |f(x_k, t, 3)| &= \left| \frac{pm_3}{N} + pm_3 \frac{x_k + 1}{\sum_{i=1}^N (x_i + 1)} \right| \leq pm_3(N+2) |x_k| \\
 |f(x_k, t, 4)| &= 0 \leq |x_k|
 \end{aligned}$$

If we set  $K = \max \left\{ \frac{rm_1}{(pm_3 - rm_1)t_0}, \frac{qm_2}{2}, pm_3(N+2), 1 \right\}$ ,

we have

$$|f(x_k, t, i)| \leq K |x_k|, \quad t \in [t_0, \infty)$$

for all  $(x_k, t, i) \in R^+ \times R^+ \times S$  at any time and state.

**Remark 3** Lemma 2 provides a feasible condition for the almost surely exponential stability of the trivial solution. (10) is valid for any node at any time and state. For any node, we assume that the absolute value of the linkage variant rate is not more than a constant multiple of links' absolute value. This is one focus of this paper that the trivial solution of (1) is also almost surely exponentially stable.

**Definition 1** Assume that there is a positive constant  $K$  such that  $|f(x_k, t, i)| \leq K |x_k|$  for all  $(x_k, t, i) \in R^+ \times R^+ \times S$ . Let  $\varphi > 0$  and  $\lambda > 0$ . If

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log \left( E |x_k(t)|^\varphi \right) \leq -\lambda$$

then the trivial solution of (1) is almost surely exponentially stable.  $\blacksquare$

**Theorem 1** For the linkage model one, if

$$\begin{vmatrix}
 -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\
 -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\
 -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\
 -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44}
 \end{vmatrix} > 0, \quad (11)$$

where  $\alpha_1 = -\frac{2}{N^2 - N} rm_1, \alpha_2 = \frac{1}{N} qm_2, \alpha_3 = \frac{3}{N} pm_3$ , and

$\alpha_4 = w$ ,

then the trivial solution of (1) is almost surely exponentially stable.  $\blacksquare$

This theorem can be proved by Lemmas 1 and 2, Definition 1 and Theorem 4.6 in [10] directly. In the following, we will simply verify this theorem.

We can obtain  $\alpha_i (i=1, 2, 3, \text{ and } 4)$  in order to meet condition (9) in Lemma 1. Moreover, we can find constant  $K$  to make condition (10) satisfied in Lemma 2.

It is known that  $\alpha_1 < 0$  and  $\alpha_i > 0 (i=2, 3, \text{ and } 4)$ .

Thus the trivial solution of (1) is almost surely exponentially stable only if

$$\begin{vmatrix}
 -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\
 -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\
 -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\
 -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44}
 \end{vmatrix} > 0. \quad \blacksquare$$

**Remark 4** In Theorem 1 on basis of Lemmas 1 and 2, we obtain the novel result that if some conditions are met, the trivial solution of (1) for model one will be almost surely exponentially stable using Markov jump theory.

This theorem also tells us that under conditions (9) and (10), the trivial solution is almost surely exponentially stable as long as we can find the transition rate to satisfy condition (11) for the system.

From the above arguments, for the first model, we find the sufficient conditions of almost surely exponential stability of the integrated system made up of four states. In other words, the solutions of the system will be almost surely exponentially stable if the links and linkage variant rates of any node meet some conditions. And then, we cite an example that is almost surely exponentially stable but

their conditions in Theorem 1 are not met.

Assume  $\Gamma = (0)_{4 \times 4}$ , evidently the trivial solution of (1) is almost surely exponentially stable, but

$$\begin{vmatrix} -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\ -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\ -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\ -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44} \end{vmatrix} = 0. \text{ Condition (11) is not}$$

met. So it is only the sufficient condition.

### 3.2 Specific Linkage Model Two

In the second model, new links are preferentially deployed evenly, while the more links the node is associated with, the higher the probability that the node removes a link. According to this case and [9],

$$Q_2(x_k) = Q_3(x_k) = \frac{1}{\sum_{i=1}^N \frac{1}{x_i + 1}} \approx \frac{1}{\sum_{i=1}^N \frac{1}{E(x_i) + 1}} = \frac{1}{x_k + 1} = \frac{2(pm_3 - rm_1)t + N}{N^2(x_k + 1)}$$

where  $E(x_i)$  denotes the expectation of  $x_i$ .

$$Q_1(x_k) = \frac{x_k}{\sum_{\text{all node } i} x_i} = \frac{x_k}{2(pm_3 - rm_1)t} = A_k. \quad (12)$$

**Theorem 2** For the linkage model two, if

$$\begin{vmatrix} -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\ -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\ -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\ -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44} \end{vmatrix} > 0 \quad (13)$$

where

$$\alpha_1 = -\frac{2}{N^2 - N} rm_1, \quad \alpha_2 = \frac{2N - 1}{2N} qm_2, \quad \alpha_3 = \frac{2N + 1}{2N} pm_3, \quad \alpha_4 = w, \text{ where } pm_3 - rm_1 > 0, \text{ then the trivial solution of (1) is almost surely exponentially stable. } \blacksquare$$

**Proof** The process of this proof is similar to Theorem 1.  $\blacksquare$

Following model one, (9) and (10) hold. For the specific linkage model two, we also find the sufficient conditions of almost surely exponential stability of the integrated system made up of four modes as long as the transition rate satisfies the condition (13).

### 3.3 Specific Linkage Model Three

In the model three, the probability of removing links is relative to the connectivity conditions of the system. According to this case and [9],

$$Q_2(x_k) = Q_3(x_k) = \frac{x_k + 1}{\sum_{\text{all node } i} (x_i + 1)} = \frac{x_k + 1}{2(pm_3 - rm_1)t + N},$$

$$Q_1(x_k) = \frac{x_k}{\mu N + \sum_{\text{all node } i} x_i} = \frac{x_k}{\mu N + 2(pm_3 - rm_1)t} = A_k \quad (14)$$

where  $\mu$  is the modulating factor.

**Theorem 3** For the model linkage three, if

$$\begin{vmatrix} -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\ -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\ -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\ -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44} \end{vmatrix} > 0 \quad (15)$$

where

$$\alpha_1 = -\frac{2}{\mu N + N^2 - N} rm_1, \alpha_2 = \frac{1}{N} qm_2, \alpha_3 = \frac{3}{N} pm_3,$$

$\alpha_4 = w$ , then the trivial solution of (1) is almost surely exponentially stable.  $\blacksquare$

**Proof** We obtain  $\alpha_2$  and  $\alpha_3$  in a similar way with Lemma 1, and argue  $\alpha_1$  in the following.

It is known that the maximum of  $(pm_3 - rm_1)t$  is

$$\frac{N^2 - N}{2} \text{ in a network with } N \text{ nodes. Thus}$$

$$x_k f(x_k, t, 3) = x_k \left[ -\frac{2rm_1 x_k}{\mu N + 2(pm_3 - rm_1)t} \right] \leq -\frac{2rm_1}{\mu N + N^2 - N} x_k^2$$

Thus we can have  $\alpha_1 = -\frac{2}{\mu N + N^2 - N} rm_1$ .

To sum up the above arguments, let

$$\alpha_1 = -\frac{2}{\mu N + N^2 - N} rm_1, \alpha_2 = \frac{1}{N} qm_2, \alpha_3 = \frac{3}{N} pm_3, \text{ and } \alpha_4 = w.$$

The rest of this proof is similar to Theorem 1.  $\blacksquare$

Following model one, if (9) and (10) hold, we also find the sufficient conditions of almost surely exponential stability of the integrated system made up of four states for the specific linkage model three as long as the transition rate satisfies condition (15).

## 4. Numerical Examples

In this section we shall discuss some examples to illustrate our conclusions.

### Example 1.

For the model one, let (9) and (10) hold. Assume

$$\Gamma = \begin{pmatrix} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 2 & -4 & 1 \\ 1 & 2 & 0 & -3 \end{pmatrix} \quad (16)$$

such that

$$\frac{32}{N^2 - N} rm_1 - \frac{20}{N} qm_2 - \frac{27}{N} pm_3 - 3w > 0 \quad (17)$$

by Theorem 1, where  $r + q + p + w = 1$ .

Then the trivial solution of (1) is almost surely

exponentially stable.

**Proof** Assume

$$\begin{pmatrix} -\alpha_1 & -\gamma_{12} & -\gamma_{13} & -\gamma_{14} \\ -\alpha_2 & -\gamma_{22} & -\gamma_{23} & -\gamma_{24} \\ -\alpha_3 & -\gamma_{32} & -\gamma_{33} & -\gamma_{34} \\ -\alpha_4 & -\gamma_{42} & -\gamma_{43} & -\gamma_{44} \end{pmatrix} = \begin{pmatrix} \frac{2}{N^2-N}rm_1 & -1 & -1 & 0 \\ -\frac{1}{N}qm_2 & 2 & -1 & 0 \\ -\frac{3}{N}pm_3 & -2 & 4 & -1 \\ -w & -2 & 0 & 3 \end{pmatrix} > 0$$

such that  $\frac{32}{N^2-N}rm_1 - \frac{20}{N}qm_2 - \frac{27}{N}pm_3 - 3w > 0$ .

It is known by Theorem 1 the trivial solution of (1) is almost surely exponentially stable. ■

**Remark 5** It is easy to verify that if (17) is met, the solutions are almost surely exponentially stable for the transition rate matrix (16). In the other words, there is an inequality to subject to any transition rate matrix in the integrated system.

Likewise, we will discuss a typical case in the following.

**Example 2.**

Let (9) and (10) hold. Assume

$$\Gamma = \begin{pmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{pmatrix}, \quad (18)$$

such that

$$\frac{2}{N^2-N}rm_1 - \frac{1}{N}qm_2 - \frac{3}{N}pm_3 - w > 0 \quad (19)$$

by Theorem 1, where  $r + q + p + w = 1$ ,

then the trivial solution of (1) is almost surely exponentially stable.

**Remark 6** This typical case shows that the four modes of the integrated system are probability-preserving transformation.

In the following, we will make the numerical analysis with Theorems 2 and 3.

On the basis of example 2, let (9) and (10) hold. Let (18) hold, such that

$$\frac{4}{N^2-N}rm_1 - \frac{2N-1}{N}qm_2 - \frac{2N+1}{N}pm_3 - 2w > 0$$

by Theorem 2, where  $r + q + p + w = 1$ ,

Then the trivial solution of (1) is almost surely exponentially stable.

Likewise, let (9) and (10) hold. Let (18) hold, such that

$$\frac{2}{\mu N + N^2 - N}rm_1 - \frac{1}{N}qm_2 - \frac{3}{N}pm_3 - w > 0$$

by Theorem 3, where  $r + q + p + w = 1$ ,

then the trivial solution of (1) is almost surely exponentially stable.

## 5. CONCLUSION

In this paper, we analyze the linkage for special cases in mobile ad hoc networks by Markov jump theory, and obtain the result that if the proposed conditions are satisfied, the solutions of these three specific cases are almost surely exponentially stable. It is a challenge to analyze the linkage for a general model system, which is our future work.

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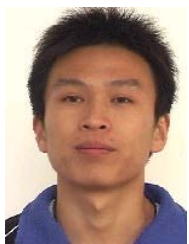
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